ЗАКАРПАТСЬКИЙ УГОРСЬКИЙ ІНСТИТУТ ІМЕНІ ФЕРЕНЦА РАКОЦІ II
II. RÁKÓCZI FERENC KÁRPÁTALJAI MAGYAR FŐISKOLA

Кафедра математики та інформатики
Matematika és Informatika Tanszék

«ENGLISH FOR MATHEMATICIANS»
(МЕТОДИЧНІ ВКАЗІВКИ ДЛЯ ПРАКТИЧНИХ ЗАНЯТЬ)

(для студентів 4-го курсу спеціальності 014 Середня освіта (Математика)

ENGLISH FOR MATHEMATICIANS

(Módszertani utmutató gyakorlati foglalkozásokhoz)

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Посібник з англійської мови для математиків призначений для студентів IV курсу Закарпатського угорського інституту імені Ференца Ракоці спеціальності 014 Середня освіта (математика) денної та заочної форми навчання з метою організації практичної роботи з курсу "Іноземна мова за професійним спрямуванням".

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Az English for mathematicians a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola, IV. éves, matematika szakos, nappali és levelezős hallgatóinak készült, a Szakmai idegen nyelv c. tantárgy alaposabb tanulmányozásának és elsajátításának megkönnyítése céljából.

Ez a jegyzet elsősorban matematika szakos hallgatók számára készült, de hasznos lehet mindazok számára, akik bármely más szakon tanulnak matematikát.

A jegyzet fejezetekre van osztva, és minden fejezet két részből áll. Az első részben röviden egy téma let kidolgozva, melyben általában fogalmak és alapelvek vannak bemutatva. A második rész pedig gyakorlati feladatokból áll, amelyek kidolgozása segít a fogalmak megértésében és angol nyelvű elsajátításában.

Az oktatási folyamatban történő felhasználását jóváhagyta
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Matematika és Informatika Tanszéke
(2024. augusztus 13, 1. számú jegyzőkönyv).

Megjelentetésre javasolta a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola
Minőségbiztosítási Tanácsa
(2024. augustzus 26 , 22. számú jegyzőkönyv).

Elektronikus formában (PDF fájlformátumban) történő kiadásra javasolta
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(2024. augustzus 27 , 7. számú jegyzőkönyv).

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Part I

ARITHMETIC AND ALGEBRA

LESSON I

ARITHMETIC

Arithmetic, the oldest and most basic branch of mathematics, deals with numbers and the basic operations: addition, subtraction, multiplication, and division. The history of arithmetic is intertwined with the history of human civilization and can be traced back to ancient times. Around 3000 BCE, the Sumerians in Mesopotamia developed a numeral system that could represent large numbers and perform calculations. The Babylonians later improved this system around 1900 BCE, using a base-60 (sexagesimal) system, which is still used today in measuring time and angles. In ancient Egypt, around 3000 BCE, a decimal system based on ten was used. Egyptians developed methods for basic arithmetic operations, particularly for taxation and trade, and recorded their mathematical knowledge on papyrus scrolls.

Greek mathematicians like Pythagoras and Euclid made significant contributions to arithmetic. Euclid’s "Elements," written around 300 BCE, included many arithmetic concepts that laid the foundation for number theory. In ancient China, mathematicians developed their own numerical system and arithmetic techniques independently. "The Nine Chapters on the Mathematical Art," written around 100 BCE, contained methods for various arithmetic operations, including fractions and square roots.

During the Middle Ages, the Hindu-Arabic numeral system, which originated in India around the 5th century, spread to the Islamic world and later to Europe. This system, with its use of zero and positional notation, revolutionized arithmetic. Indian mathematicians like Aryabhata and Brahmagupta made significant contributions to arithmetic, including the concept of zero and negative numbers. Islamic scholars translated and expanded upon Greek and Indian mathematical works. Al-Khwarizmi, a Persian mathematician, wrote a book around 820 CE that introduced the Hindu-Arabic numeral system to the Islamic world and eventually to Europe. The introduction of the Hindu-Arabic numeral system to Europe by Fibonacci in his book "Liber Abaci" (1202) marked a turning point. This new system replaced the cumbersome Roman numerals and facilitated the spread of arithmetic knowledge.

The Renaissance period saw further advancements in arithmetic. European mathematicians developed new techniques and expanded the applications of arithmetic in commerce, navigation, and science. The invention of the printing press in the 15th century allowed for the widespread distribution of arithmetic texts, making mathematical knowledge more accessible. The widespread adoption of the decimal system simplified arithmetic calculations and laid the groundwork for modern mathematics.

In the modern era, arithmetic has evolved with the development of computers and digital technology. Arithmetic operations are now performed electronically, allowing for complex calculations and applications in various fields such as engineering, finance, and data science. The history of arithmetic is a testament to human ingenuity and the quest for knowledge. From ancient civilizations to the modern digital age, arithmetic has been a fundamental tool for understanding and shaping the world.

Vocabulary Notes

arithmetic - aritmetika	fractions - törtek
mathematics - matematika	square roots - négyzetgyökök
addition - összeadás	middle ages - középkor
subtraction - kivonás	arithmetic techniques - aritmetikai módszerek
multiplication - szorzás	zero - nulla
division - osztás	positional notation - helyiértékes számírás
ancient times - ősi idők	revolutionized - forradalmasította
Mesopotamia - Mezopotámia	negative numbers - negatív számok
sumerians - sumérok	expanded upon - kibővítették
numeral system - számrendszer	facilitated - megkönnyítette
babylonians - babiloniak	spread of knowledge - tudás terjesztése
base-60 (sexagesimal) system - hatvanas számrendszer	advancements - fejlődések
measuring time - idő mérés	applications - alkalmazások
angles - szögek	commerce - kereskedelem
ancient egypt - ókori egyiptom	navigation - navigáció
decimal system - tízes számrendszer	science - tudomány
taxation - adózás	distribution - terjesztés
trade - kereskedelem	accessible - elérhető
papyrus scrolls - papirusztekercsek	data science - adatelemzés
greek mathematicians - görög matematikusok	testament - tanúsítvány
Pythagoras - Püthagorasz	ingenuity - leleményesség
Euclid - Euklidész	quest for knowledge - tudásszomj
Elements - „Elemek”	digital age - digitális korszak
number theory - számelmélet	fundamental tool - alapvető eszköz

Exercises

I. *Answer the following questions on the text:*

1. What is arithmetic and what are its basic operations?
2. Which ancient civilization first developed a numeral system around 3000 BCE?
3. How did the Babylonians improve the numeral system, and what is this system still used for today?
4. What system did the ancient Egyptians use for their arithmetic operations around 3000 BCE?

5. Who were some of the notable Greek mathematicians that contributed to arithmetic?
6. What significant contributions did Indian mathematicians like Aryabhata and Brahmagupta make to arithmetic?
7. How did the Hindu-Arabic numeral system revolutionize arithmetic during the Middle Ages?
8. What role did Al-Khwarizmi play in the spread of the Hindu-Arabic numeral system?
9. How did the invention of the printing press during the Renaissance period impact the distribution of arithmetic knowledge?
10. How has arithmetic evolved in the modern era with the development of computers and digital technology?

II. *Mark the following statements as true or false based on the information in the passage*:

1. The history of arithmetic began in ancient Greece.
2. The Hindu-Arabic numeral system uses zero and positional notation.
3. The printing press hindered the spread of arithmetic knowledge.
4. Modern computers rely on electronic arithmetic operations.

III. *Match the following terms with their definitions related to arithmetic*:

1. Fraction
2. Square root
3. Algorithm
4. Numeral system
5. Decimal system

(a) A system for representing numbers using the digits 0-9 and place value.
(b) A collection of symbols used to represent numbers.
(c) A set of step-by-step instructions for performing a calculation.
(d) A number that represents a part of a whole.
(e) A number that, when multiplied by itself, equals another number.

IV. *Complete the following sentences with appropriate words from the passage*:

1. The Sumerians and Babylonians developed early numeral systems for representing large numbers and performing _______.
2. Euclid's "Elements" laid the foundation for the study of _______.
3. The book "The Nine Chapters on the Mathematical Art" included methods for various _______ operations.
4. The invention of the _______ in the 15th century made arithmetic texts more accessible.

V. *Research another historical numeral system (e.g., Roman numerals, Egyptian hieroglyphs) and explain how it functioned*.

LESSON II

ADDITION

Addition is one of the most fundamental operations in arithmetic and mathematics. It forms the basis for many more complex calculations. Understanding addition is crucial because it is used in everyday life and in advanced mathematical concepts. This text will explain what addition is, its properties, and how to perform it.

What is Addition?

Addition is the process of combining two or more numbers to find their total or sum. For example, if you have two apples and you get three more, you now have a total of five apples. In mathematical terms, this is expressed as 2 + 3 = 5. Here, the numbers being added (2 and 3) are called addends, and the result (5) is called the sum.

How to Perform Addition

To perform addition, follow these simple steps:

1. Write the numbers you want to add, one below the other, aligning the digits by place value (units, tens, hundreds, etc.).
2. Start adding from the rightmost digit (units place).
3. If the sum of a column is 10 or more, write down the rightmost digit of the sum and carry over the left digit to the next column.
4. Continue this process for each column, moving to the left.
5. Write down the final sum once all columns have been added.

Example:

$$\begin{array}{r} 47 \\ + \\ \underline{35} \\ 82 \end{array}$$

Start with the units place: 7 + 5 = 12. Write down 2 and carry over 1 to the tens place. Then, add the tens place: 4 + 3 + 1 (carry over) = 8. So, 47 + 35 = 82.

Addition has several important properties that help us understand and perform the operation more effectively:

1. **Commutative Property**: The order in which numbers are added does not affect the sum. For example, 4 + 5 is the same as 5 + 4, and both equal 9.
2. **Associative Property**: When adding three or more numbers, the grouping does not affect the sum. For example, (2 + 3) + 4 is the same as 2 + (3 + 4), and both equal 9.

3. **Identity Property**: Adding zero to any number does not change the value of that number. For example, 7 + 0 equals 7.
4. **Distributive Property**: This property combines addition and multiplication, showing that a number multiplied by the sum of two others is the same as multiplying the number by each addend and then adding the results. For example, 2 * (3 + 4) is the same as (2 * 3) + (2 * 4), and both equal 14.

Addition is not only performed in the base-10 (decimal) system but also in other number systems, such as binary (base-2), octal (base-8), and hexadecimal (base-16). Each system has its own rules for carrying over digits, but the basic principles remain the same.

Addition is a fundamental arithmetic operation that everyone should master. Its properties ensure consistency and reliability, making it indispensable in both simple everyday tasks and complex scientific calculations. By understanding and practicing addition, you build a strong foundation for all other areas of mathematics.

Vocabulary Notes

addition - összeadás fundamental - alapvető operations - műveletek complex calculations - bonyolult számítások properties - tulajdonságok perform - végrehajtani process - folyamat combining - kombinálás total - összeg sum - összeg result - eredmény aligning - igazítani digits - számjegyek place value - helyi érték units place - egységek helye rightmost digit - jobb szélső számjegy carry over - átvinni column - oszlop	final sum - végső összeg commutative property - kommutatív tulajdonság affect - befolyásol equal - egyenlő associative property - asszociatív tulajdonság grouping - csoportosítás identity property – egységelem létezésének tulajdonsága adding zero - nulla hozzáadása distributive property - disztributív tulajdonság rules - szabályok principles - alapelvek ensure - biztosítani consistency - következetesség reliability - megbízhatóság indispensable - nélkülözhetetlen

Exercises

I. *Answer the following questions on the text:*

1. What is addition, and why is it considered fundamental in arithmetic and mathematics?
2. Can you explain the process of addition using an example from the text?
3. What are addends and what is the sum in the context of addition?
4. Describe the steps involved in performing addition as outlined in the text.
5. What properties of addition are mentioned in the text, and how do they influence the operation?

6. How does addition relate to other arithmetic operations, such as multiplication, according to the distributive property?

II. *Give the Hungarian for the following word combinations. Use them in sentences of your own:* Fundamental operations, forms the basis for, crucial in, everyday life, advanced mathematical concepts, process of combining, total or sum, mathematical terms, called addends, result is called the sum, perform addition, simple steps, aligning the digits, place value, rightmost digit, carry over, final sum, important properties, understand and perform, Commutative Property, Associative Property, Identity Property, Distributive Property, base-10 (decimal) system, basic principles, fundamental arithmetic operation, ensure consistency and reliability, indispensable in, strong foundation.

III. *What is the main purpose of addition*?

(a) To simplify complex fractions
(b) To find the total by combining numbers
(c) To measure angles and distances
(d) To represent numbers using symbols

IV. *In the equation* $4 + 7 = 11$, *what are the numbers being added called*?

(a) Digits
(b) Coefficients
(c) Addends
(d) Variables

V. *When performing addition, how do we handle a sum greater than* 9 *in a specific column*?

(a) Ignore the extra digit and move to the next column.
(b) Write down the entire sum in that column.
(c) Carry over the leftmost digit to the next column and write down the rightmost digit in the current column.
(d) Subtract the unit digit from 10 and write it down.

VI. *Which property of addition states that the order of adding numbers doesn't affect the sum*?

(a) Associative property
(b) Commutative property
(c) Distributive property
(d) Identity property

VII. *How is the concept of addition applied in other number systems like binary or hexadecimal*?

(a) The symbols used for numbers change, but the core principles remain the same.
(b) Addition becomes significantly more complex and requires different rules.
(c) Addition is only applicable in the base-10 system.
(d) The concept of carrying over doesn't exist in other number systems.

LESSON III
SUBTRACTION

Subtraction is another fundamental operation in arithmetic and mathematics. It is essential for many aspects of daily life and advanced mathematical concepts. This section will explain what subtraction is, its properties, and how to perform it.

Subtraction is the process of finding the difference between two numbers by removing a quantity from another. For example, if you have five apples and you give away two, you are left with three apples. In mathematical terms, this is expressed as 5 - 2 = 3. Here, the number being subtracted (2) is called the subtrahend, the number from which it is subtracted (5) is called the minuend, and the result (3) is called the difference.

How to Perform Subtraction

To perform subtraction, follow these simple steps:

1. Write the Numbers: Write the numbers you want to subtract, one below the other, aligning the digits by place value (units, tens, hundreds, etc.).
2. Subtract from the Rightmost Digit: Start subtracting from the rightmost digit (units place).
3. Borrow if Necessary: If the digit in the minuend is smaller than the corresponding digit in the subtrahend, borrow 1 from the next higher place value.
4. Continue to the Left: Continue this process for each column, moving to the left.
5. Write the Final Difference: Write down the final difference once all columns have been subtracted.

Example:

$$\begin{array}{r} 53 \\ - \\ \underline{\quad 27} \\ 26 \end{array}$$

Start with the units place: 3 - 7 is not possible without borrowing. Borrow 1 from the tens place, making the subtraction 13 - 7 = 6. Then, adjust the tens place: 4 - 2 = 2. So, 53 - 27 = 26.

Subtraction has several important properties that help us understand and perform the operation more effectively:

Non-Commutative Property: The order in which numbers are subtracted affects the difference. For example, 5 - 3 is not the same as 3 - 5. While 5 - 3 equals 2, 3 - 5 equals -2.

Non-Associative Property: When subtracting three or more numbers, the grouping affects the difference. For example, (8 - 3) - 2 is not the same as 8 - (3 - 2). The first expression equals 3, while the second equals 7.

Identity Property: Subtracting zero from any number does not change the value of that number. For example, 6 - 0 equals 6.

Inverse Operation: Subtraction is the inverse operation of addition. This means that if a + b = c, then c - b = a and c - a = b.

Vocabulary Notes

subtraction - kivonás difference - különbség quantity - mennyiség remove - eltávolít subtrahend - kivonandó minuend - kisebbítendő perform - végrehajt	borrow - kölcsönvesz final difference - végső különbség non-commutative property - nem kommutatív tulajdonság non-associative property - nem asszociatív tulajdonság inverse operation - inverz művelet

Exercises

I. *Answer the following questions on the text*:

1. What is subtraction?
2. What are the names of the numbers involved in a subtraction operation?
3. Describe the steps to perform subtraction.
4. What do you do if the digit in the minuend is smaller than the corresponding digit in the subtrahend?
5. What is the result of 64 - 38, and how do you calculate it?
6. Explain the non-commutative property of subtraction with an example.
7. How does the non-associative property affect the subtraction of three or more numbers?
8. What is the identity property of subtraction?
9. How is subtraction related to addition as an inverse operation?

II. *Give the Hungarian for the following word combinations. Use them in sentences of your own:*

Process of finding the difference, removing a quantity, mathematical terms, subtracted (subtrahend), subtracted from (minuend), result (difference), to perform subtraction, aligning the digits by place value, units, tens, hundreds, subtract from the rightmost digit, borrow if necessary, corresponding digit, borrow from the next higher place value, continue to the left, write the final difference, units place, tens place, important properties, non-commutative property, order in which numbers are subtracted, affects the difference, non-associative property, grouping affects the difference, identity property, inverse operation.

III. *Match the following terms with their definitions related to subtraction*:

1. Borrowing
2. Minuend
3. Difference
4. Subtrahend

(a) The number being subtracted.

(b) The result of the subtraction.

(c) The number from which something is subtracted.

(d) The process of taking a value from the next place value column in the minuend.

IV. *Indicate whether the following statements about subtraction are true or false based on the information in the passage*:

1. The order of subtracting numbers doesn't affect the difference.
2. Subtracting zero from a number always results in zero.
3. Subtraction is the same operation as addition performed backwards.
4. Borrowing is always necessary when performing subtraction.

V. *Complete the following sentences with appropriate words from the passage*:

(a) In the equation $8 - 3 = 5, 8$ is called the _______ and 3 is called the _______.

(b) When a digit in the minuend is smaller than the corresponding digit in the subtrahend, we need to _______.

VI. *Subtraction isn't just about numbers on a page. Explain how subtraction is used in a specific field that interests you (e.g., sports statistics, cooking measurements, budgeting). Provide an example to illustrate your point.*

VII. *We learned that the order of subtraction matters. Explain why this property wouldn't apply to addition. Create a mathematical example to demonstrate the difference between subtraction and addition in terms of order.*

VIII. *Design a simple subtraction game that can be played with a partner or by yourself. The game should involve using subtraction skills in a fun and interactive way. Describe the rules and how the game would work.*

LESSON IV
MULTIPLICATION

Multiplication is one of the core operations in arithmetic and mathematics. It is used in a wide range of everyday activities as well as in advanced mathematical concepts. This section will explain what multiplication is, its properties, and how to perform it.

Multiplication is the process of combining groups of equal sizes to find their total. It is essentially repeated addition. For instance, if you have three groups of four apples each, you can find the total number of apples by multiplying 3 by 4. In mathematical terms, this is expressed as $3 \times 4 = 12$. Here, the numbers being multiplied (3 and 4) are called factors, and the result (12) is called the product.

How to Perform Multiplication

To perform multiplication, follow these steps:

1. Write the Numbers: Write the numbers you want to multiply, aligning them by place value (units, tens, hundreds, etc.).
2. Multiply the Rightmost Digit: Start by multiplying the rightmost digit of the bottom number by each digit of the top number.
3. Carry Over if Necessary: If the product of the digits is 10 or more, write down the rightmost digit and carry over the left digit to the next column.
4. Continue the Process: Move to the next digit of the bottom number and repeat the process, adding the results for each digit.
5. Write the Final Product: Sum the results of each row to get the final product.

Example

23

×

<u>15</u>

115 (23 × 5)

<u>230 (23 × 10)</u>

345

Start by multiplying the rightmost digit of the bottom number (5) by each digit of the top number: $5 \times 3 = 15$, write down 5, and carry over 1. Then, $5 \times 2 = 10$, plus the carried over 1 equals 11. Write down 11. Next, multiply the second digit of the bottom number (1, which represents 10) by the top number: $10 \times 3 = 30$, write down 0, and carry over 3. Then, $10 \times 2 = 20$, plus the carried over 3 equals 23. Write down 230. Finally, add the results: $115 + 230 = 345$.

Properties of Multiplication

Commutative Property: The order of the factors does not affect the product. For example, 3 × 4 is the same as 4 × 3, and both equal 12.

Associative Property: The grouping of factors does not affect the product. For example, (2 × 3) × 4 is the same as 2 × (3 × 4), and both equal 24.

Distributive Property: This property links multiplication and addition, showing that a number multiplied by the sum of two others is the same as multiplying the number by each addend and then adding the results. For example, 2 × (3 + 4) is the same as (2 × 3) + (2 × 4), and both equal 14.

Identity Property: Multiplying any number by one does not change the value of that number. For example, 6 × 1 equals 6.

Zero Property: Multiplying any number by zero results in zero. For example, 7 × 0 equals 0.

Vocabulary Notes

multiplication – szorzás core operations – alapműveletek repeated addition – ismételt összeadás factors – tényezők product – szorzat units place – egyes helyi érték tens place – tizes helyi érték hundreds place – százas helyi érték	rightmost digit – jobb szélső számjegy carry over – átvitel order of the factors – a tényezők sorrendje grouping of factors – a tényezők csoportosítása addends – összeadandók

Exercises

I. *Answer the following questions on the text*:

1. How is multiplication defined in terms of repeated addition?
2. In the example given, how do you find the total number of apples when you have three groups of four apples each?
3. What are the numbers being multiplied in a multiplication problem called?
4. What is the result of a multiplication problem called?
5. What is the first step in performing multiplication?
6. How do you handle the rightmost digit when performing multiplication?
7. What should you do if the product of the digits is 10 or more during multiplication?
8. How do you continue the multiplication process after handling the rightmost digit?
9. What are the properties of multiplication described in the text?

II. *Give the Hungarian for the following word combinations. Use them in sentences of your own*: Core operations, repeated addition, factors and product, to perform multiplication, aligning them by place value, commutative property, order of the factors, associative property, grouping of factors, distributive property, links multiplication and addition, identity property, multiplying any number by one, zero property, multiplying any number by zero.

III. *Match the following terms with their definitions related to multiplication*:

1. Factors
2. Product
3. Distributive Property
4. Zero Property

(a) Multiplying by zero results in zero.
(b) A property that connects multiplication and addition.
(c) The result of multiplying two or more numbers.
(d) Numbers being multiplied.

IV. *Indicate whether the following statements about multiplication are true or false based on the information in the passage*:

1. Multiplication is the same as repeated subtraction.
2. The order in which you multiply factors doesn't affect the product.
3. Multiplying any number by 1 leaves the number unchanged.
4. The sum of two numbers multiplied by a factor is the same as multiplying each number by the factor and adding the products.

V. *Multiplication isn't just used with whole numbers. Explain how multiplication is used with fractions or decimals. Provide an example to illustrate your concept.*

VI. *Explore how multiplication is used in a specific field that interests you (e.g., physics, cooking, sports statistics). Provide an example to illustrate your point. Can you think of any surprising applications of multiplication in everyday life?*

VII. *Complete the following sentences with appropriate words from the passage*:

1. Multiplication is the process of combining groups of equal sizes to find their _____. It is essentially repeated _____.
2. In the equation $3 \times 4 = 12$, 3 and 4 are called _____, and the result (12) is called the _____.
3. When performing multiplication, we start by multiplying the rightmost digit of the bottom number by each digit of the top number. If the product of the digits is 10 or more, we write down the rightmost digit and carry over the left digit to the next _____.
4. The ____ property of multiplication states that the order of the factors does not affect the product. For example, 3×4 is the same as 4×3, and both equal _____.
5. The ____ property of multiplication links multiplication and addition, showing that a number multiplied by the sum of two others is the same as multiplying the number by each ____ and then adding the results. For example, $2 \times (3 + 4)$ is the same as $(2 \times 3) + (2 \times 4)$, and both equal _____.
6. Multiplying any number by ____ does not change the value of that number. For example, 6×1 equals _____.
7. Multiplying any number by ____ results in zero. For example, 7×0 equals _____.

LESSON V
DIVISION

Division is a fundamental arithmetic operation used to distribute or partition a quantity into equal parts or groups. It is the inverse operation of multiplication. This section will explain what division is, its properties, and how to perform it.

Division is the process of splitting a number into equal parts or finding out how many times one number is contained within another. For example, if you have 12 cookies and want to share them equally among 3 friends, you would divide 12 by 3. In mathematical terms, this is expressed as

$$\frac{12}{3} = 4.$$

Here, 12 is called the *dividend*, 3 is the *divisor*, and 4 is the *quotient*.

To perform division, follow these steps:

1. Set Up the Division:

Write the dividend (the number to be divided) inside the division bracket, and the divisor (the number by which to divide) outside the bracket.

$\underline{\ \ 12}$

3

2. Divide the Digits:

Start dividing the leftmost digit of the dividend by the divisor. Write down the result (quotient) above the dividend.

$\underline{\ \ 12}$

3 | 4

3 goes into 12 exactly 4 times.

3. Multiply and Subtract:

Multiply the divisor (3) by the quotient (4) and write the result below the dividend. Subtract this result from the current portion of the dividend.

$\underline{\ \ 12}$

3 | 4

$\underline{\ \ -12}$

0

There is no remainder, so the division is exact.

Write the Final Quotient:

Write down the quotient (4) as the result of the division.

$$\begin{array}{l} \underline{\ \ 12} \\ 3 \mid 4 \\ \underline{\ \ -12} \\ \ \ \ 0 \end{array}$$

Quotient: 4

Properties of Division

Division by Zero: Division by zero is undefined and cannot be performed.

Identity Property: Dividing any number by one leaves the number unchanged. For example, $8 \div 1 = 8$.

Inverse Operation: Division is the inverse of multiplication. If $a \times b = c$, then $c \div b = a$ and $c \div a = b$.

Non-commutative Property: The order of numbers in division affects the result. For example, $12 \div 3$ is not the same as $3 \div 12$.

Non-associative Property: Division is not associative. For example, $(12 \div 3) \div 2$ is not the same as $12 \div (3 \div 2)$.

Vocabulary Notes

division - osztás quotient - hányados	dividend - osztandó divisor - osztó

Exercises

I. *Answer the following questions on the text*:

1. How is division related to multiplication?
2. Can you explain the process of division with an example?
3. What are the key terms used in division, such as dividend, divisor, and quotient?
4. Why is division by zero undefined?
5. How does division relate to the identity property of arithmetic?
6. Why is division considered the inverse operation of multiplication?
7. How does the order of numbers affect the result in division?
8. Why is division not associative? Provide an example.
9. What are some everyday scenarios where division is used?

II. *Give the Hungarian for the following word combinations. Use them in sentences of your own*: Inverse operation of multiplication, distribute or partition a quantity, dividend, divisor, quotient, set up the division, divide the digits, multiply and subtract, exact division.

III. *Match the following terms with their definitions related to division*:

1. Dividend
2. Divisor
3. Quotient
4. Remainder

(a) The amount left over after an exact division isn't possible.

(b) The number being divided.

(c) The answer we get after dividing.

(d) The number we divide by.

IV. *Indicate whether the following statements about division are true or false based on the information in the passage*:

1. Division is the same as splitting something into pieces of different sizes.
2. The number we divide by is called the divisor.
3. The result of a division is always a whole number.
4. Division is the opposite operation of multiplication.

V. *Complete the following sentences with appropriate words from the passage*:

1. In the equation $15 \div 3 = 5, 15$ is called the ________, 3 is called the ________, and 5 is called the ________.
2. Dividing a number by 1 always results in the original ________.

VI. *Division isn't just used with whole numbers. Explain how division can be used with fractions or decimals. Provide an example to illustrate your concept*.

LESSON VI
DECIMAL AND COMMON FRACTION

Decimals and common fractions are fundamental concepts in mathematics, essential for understanding and manipulating numerical values in various contexts.

Decimals are numerical values that include a decimal point to denote parts of a whole number beyond the integer. For example, 3.5 represents three units and five tenths. Decimals can also represent more precise values, such as 3.75, which denotes three units, seven tenths, and five hundredths. The number 34.76 is read aloud as “thirty four point seven six”.

Common fractions express parts of a whole or a collection. They consist of a numerator (the top number) and a denominator (the bottom number), indicating how many parts of the whole are taken. For instance:

$\frac{1}{2}$ represents one part out of two equal parts,

$\frac{3}{4}$ represents three parts out of four equal parts,

$\frac{5}{8}$ represents five parts out of eight equal parts.

Comparison and Relationship

Decimals and common fractions can represent the same values but are presented in different forms. Understanding their equivalence and conversion between them is crucial in mathematical operations and applications. For example:

$\frac{1}{2}$ is equivalent to 0.5,

$\frac{3}{4}$ is equivalent to 0.75,

$\frac{2}{5}$ is equivalent to 0.4.

To convert a fraction to a decimal, divide the numerator by the denominator. Conversely, to convert a decimal to a fraction, identify the place value of the decimal and write it as a fraction. For example:

Convert $\frac{3}{8}$ to a decimal: $3 \div 8 = 0.375$.

Convert 0.2 to a fraction: 0.2 is $\frac{2}{10}$, which simplifies to $\frac{1}{5}$.

Operations

Addition and Subtraction:

To add or subtract decimals, align the decimal points and proceed as with whole numbers. For example, $1.25 + 0.75 = 2.00$.

To add or subtract fractions, ensure the denominators are the same before combining the numerators. For example, $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$.

Multiplication:

Multiply decimals as whole numbers, then count the total number of decimal places in both factors to place the decimal point in the product. For example, $0.6 \times 0.7 = 0.42$.

Multiply fractions by multiplying the numerators and denominators. For example, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

Division:

Divide decimals by moving the decimal point to make the divisor a whole number and adjusting the dividend accordingly. For example, $4.2 \div 0.7$ becomes $42 \div 7 = 6$.

Divide fractions by multiplying by the reciprocal of the divisor. For example, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$.

Decimals: Widely used in financial transactions (currency, e.g., $12.45), measurements (length, e.g., 5.67 meters; weight, e.g., 2.5 kilograms), and scientific calculations (precision, e.g., 0.0032 grams).

Common Fractions: Applied in cooking recipes (e.g., $\frac{1}{2}$ cup of sugar), proportions (e.g., $\frac{3}{4}$ of a total amount), and everyday situations where parts of a whole are considered (e.g., splitting a pizza into $\frac{1}{8}$ slices).

Mastering decimals and common fractions is essential for mathematical proficiency in both academic and practical contexts. They provide tools for precise measurement, accurate calculation, and effective problem-solving in various disciplines. Understanding these concepts enhances mathematical skills and facilitates their application in real-world scenarios, making them indispensable in both academic and everyday settings.

Vocabulary Notes

decimals – tizedes törtek common fractions - közönséges törtek decimal point - tizedes vessző whole number - egész szám three units - három egység five tenths - öt tized three-fourths - három negyed	five-eighths - öt nyolcad numerator - számláló denominator - nevező equivalent - egyenértékű conversion - átalakítás reciprocal - reciprok

Exercises

I. *Answer the following questions on the text*:

1. How are decimals represented?
2. What does the decimal 3.5 represent?
3. How does 3.75 differ from 3.5 in terms of place value?
4. What are common fractions?
5. What do the numerator and denominator represent in a common fraction?
6. Give an example of a common fraction and explain its meaning.
7. How can decimals and common fractions represent the same values?
8. What is the decimal equivalent of the fraction 1/2?
9. How do you convert the fraction 5/8 to a decimal?
10. How do you convert a fraction to a decimal?
11. How do you convert a decimal to a fraction?
12. Convert the decimal 0.2 to a fraction.
13. What is the process to add or subtract decimals?
14. How do you add or subtract fractions?
15. Describe the method for multiplying decimals.
16. What is the process for multiplying fractions?
17. Explain the division process for decimals.
18. How do you divide fractions?
19. List some practical applications of decimals and common fractions in everyday life.

II. *Give the Hungarian for the following word combinations. Use them in sentences of your own:* Fundamental concepts in mathematics, decimal point, parts of a whole number, three units and five tenths, more precise values, three units, seven tenths, and five hundredths, common fractions, parts of a whole or a collection, numerator and denominator, one part out of two equal parts, three parts out of four equal parts, five parts out of eight equal parts, comparison and relationship, represent the same values, equivalence and conversion, reciprocal of the divisor, financial transactions, scientific calculations, cooking recipes, proportions, everyday situations, precise measurement, accurate calculation, effective problem-solving, academic and practical contexts, real-world scenarios.

III. *Read the following numbers*: $7.56;\ \ 98.69;\ \ 0.02;\ 0.5;\ \frac{3}{7};\ \frac{7}{13};\ \frac{19}{24}.$

IV. *Match the English terms with their definitions*:

1. Decimals
2. Common fractions
3. Numerator

4. Denominator
5. Convert
6. Definitions:

a) The top number of a fraction
b) Numbers that include a decimal point to denote parts of a whole number
c) To change something into a different form
d) Numbers that consist of a numerator and a denominator
e) The bottom number of a fraction

V. *Use the words from the vocabulary list to complete the sentences*:

1. ___________ are numerical values that include a decimal point.
2. A fraction consists of a ___________ and a ___________.
3. To change a fraction to a decimal, you need to ___________ the numerator by the denominator.

VI. *What does the decimal 3.75 represent*?

a) Three units and five tenths
b) Three units, seven tenths, and five hundredths
c) Thirty-seven point five

LESSON VII
ALGEBRAIC EXPRESSION

Algebraic expressions are fundamental components in mathematics, encapsulating quantities and their relationships using symbols and operators.

An **algebraic expression** is a combination of numbers, variables, and operations (such as addition, subtraction, multiplication, and division). Here are the key components of algebraic expressions:

1. **Variables**: Symbols (usually letters) that represent unknown or variable quantities (e.g., x, y, z).
2. **Constants**: Fixed values that do not change (e.g., $3, -5, \frac{1}{2}$).
3. **Coefficients**: Numbers multiplying the variables (e.g., in $5x$, 5 is the coefficient).
4. **Operators**: Symbols indicating the operations to be performed (e.g., $+, -, \times, \div$).
5. **Terms**: The individual parts of an expression separated by plus or minus signs (e.g., in $3x + 2$, $3x$ and 2 are terms).

Types of Algebraic Expressions

1. **Monomial**: An expression with a single term (e.g., $7x$ or $-3a^2b$).
2. **Binomial**: An expression with two terms (e.g., $x + 5$ or $3a - 4b$).
3. **Trinomial**: An expression with three terms (e.g., $2x^2 + 3x + 1$).
4. **Polynomial**: An expression with one or more terms, where the exponents of the variables are non-negative integers (e.g., $4x^3 - 2x^2 + x - 7$).

Operations with Algebraic Expressions

1. **Addition**: Combine like terms by adding their coefficients.
   Example: $3x + 5x = 8x$.
2. **Subtraction**: Combine like terms by subtracting their coefficients.
   Example: $7y - 2y = 5y$.
3. **Multiplication**: Multiply the coefficients and add the exponents of the variables.
   Example: $(2x)(3x^2) = 6x^3$.
4. **Division**: Divide the coefficients and subtract the exponents of the variables.
   Example: $\frac{6x^3}{2x} = 3x^2$.

5. **Factoring**: Express an expression as a product of its factors.
   Example: $x^2 - 9 = (x + 3)(x - 3)$.
6. **Expanding**: Distribute multiplication over addition.
   Example: $(x + 2)(x + 3) = x^2 + 5x + 6$.

Simplifying Algebraic Expressions

To simplify an algebraic expression means to combine like terms and perform the operations to express the expression in its simplest form.

Example: Simplify $2x + 3x - 4 + 5$.

1. Combine like terms: $2x + 3x = 5x$.
2. Combine constants: $-4 + 5 = 1$.

So, $2x + 3x - 4 + 5$ simplifies to $5x + 1$.

Translating Algebraic Expressions from Mathematical Notation to English

Understanding how to describe algebraic expressions in English is crucial for mathematicians working in an international environment. Here are some examples:

- $x + 7x$: "x plus seven"
- $3y - 4$: "three y minus four"
- $5a \times 2b$: "five a times two b"
- $\frac{6x}{3y}$: "six x divided by three y"
- $x^2 + 3x + 2$: "x squared plus three x plus two"

Vocabulary Notes

Algebraic expressions - algebrai kifejezések symbols - szimbólumok operators - operátorok variables - változók unknown - ismeretlen variable quantities - változó mennyiségek constants - állandók fixed values - rögzített értékek	non-negative integers - nemnegatív egész számok combine like terms - hasonló tagok összevonása adding coefficients - együtthatók összeadása add the exponents - a kitevők összeadása divide the coefficients - az együtthatók elosztása

coefficients - együtthatók numbers multiplying the variables - számok, amelyek megszorozzák a változókat terms - tagok individual parts - egyéni részek plus signs - pluszjelek minus signs - mínuszjelek monomial - monomiális kifejezés single term - egyetlen tag binomial - binomiális kifejezés trinomial - trinomális kifejezés polynomial - polinomiális kifejezés exponents - kitevők	factoring - tényezőkre bontás product of its factors - tényezőinek szorzata expanding - kiterjesztés simplify - egyszerűsíteni combine like terms - hasonló tagok összevonása perform the operations - végrehajtani a műveleteket simplest form - legegyszerűbb alak combine constants - állandók összevonása crucial - kulcsfontosságú squared - négyzetre emelve

Exercises

I. *Answer the following questions on the text*:

1. What are algebraic expressions in mathematics?
2. What is an algebraic expression composed of?
3. What are variables in the context of algebraic expressions? Provide examples of variables used in algebraic expressions.
4. What are constants in algebraic expressions? Provide examples of constants used in algebraic expressions.
5. What are coefficients in algebraic expressions? Provide an example of a coefficient in an algebraic expression.
6. What are terms in an algebraic expression?
7. What is a monomial and give an example?
8. What is a binomial and give an example?
9. What is a trinomial and give an example?
10. What is a polynomial, and what are its characteristics?
11. How are the exponents of the variables in a polynomial described?
12. What operation is performed when combining like terms in addition? Provide an example of adding like terms in an algebraic expression.
13. What operation is performed when combining like terms in subtraction? Provide an example of subtracting like terms in an algebraic expression.

14. How are the coefficients and exponents treated in multiplication of algebraic expressions? Provide an example of multiplying terms in an algebraic expression.
15. How are the coefficients and exponents treated in division of algebraic expressions? Provide an example of dividing terms in an algebraic expression.
16. What does factoring an algebraic expression involve? Provide an example of factoring an algebraic expression.
17. What does expanding an algebraic expression involve? Provide an example of expanding an algebraic expression.
18. What does simplifying an algebraic expression mean?

II. *Identify the components (variables, constants, coefficients, operators, and terms) in the following algebraic expression*: $4x^2 - 3xy + 7y - 5$.

III. *Classify each of the following expressions as a monomial, binomial, trinomial, or polynomial*:$6a$; $2x + 5$; $3y^2 - 4y + 1$; $4x^3 - 3x^2 + 2x - 1$.

IV. *Combine the like terms in the following expression*: $2a + 3a - a + 5$.

V. *Factor the following expression*: $x^2 - 16$.

VI. *Expand the following expression*: $(x + 3)(x - 2)$.

VII. *Translate the following algebraic expressions to English phrases*:

1. $x + 8$
2. $4y - 74$
3. $5a \times 3b$
4. $\frac{9x}{3y}$
5. $x^2 + 6x + 9$

LESSON VIII
EQUATION AND PROPORTION

Understanding equations and proportions is crucial for mathematicians, as these concepts are fundamental in solving a wide range of mathematical problems.

An equation is a mathematical statement that asserts the equality of two expressions. It consists of two sides connected by an equals sign (=). Solving an equation involves finding the value(s) of the variable(s) that make the equation true.

Types of Equations:

1. **Linear Equations**: Equations of the first degree, meaning the highest power of the variable is one. The general form is $ax + b = 0$, where a and b are constants.

   Example: $2x + 3 = 7$.

2. **Quadratic Equations**: Equations of the second degree, meaning the highest power of the variable is two. The general form is $ax^2 + bx + c = 0$, where a, b, and c are constants.

   Example: $x^2 - 4x + 4 = 0$.

3. **Polynomial Equations**: Equations involving polynomial expressions of higher degrees.

   Example: $x^3 - 2x^2 + x - 1 = 0$.

4. **Rational Equations**: Equations involving rational expressions, which are ratios of polynomials.

   Example: $\frac{x+1}{x-1} = 3$.

5. **Radical Equations**: Equations involving roots, such as square roots or cube roots.

   Example: $\sqrt{x+3} = 2$.

Solving Equations:

1. **Isolating the Variable**: Simplify the equation to get the variable alone on one side.

   Example: Solve $2x + 3 = 7$.

Subtract 3 from both sides: $2x = 4$.

Divide both sides by 2: $x = 2$.

2. **Factoring**: Express the equation in a factored form and solve for the variable.

Example: Solve $x^2 - 4 = 0$.

Factor: $(x - 2)(x + 2) = 0$.

Solve: $x = 2$ or $x = -2$.

3. **Quadratic Formula**: For quadratic equations, use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example: Solve $x^2 - 4x + 4 = 0$.

$x = \frac{4 \pm \sqrt{16 - 16}}{2} = 2$.

4. **Cross Multiplication**: Used for rational equations to eliminate the fractions.

Example: Solve $\frac{x+1}{x-1} = 3$.

Cross multiply: $x + 1 = 3(x - 1)$.

Simplify and solve: $x + 1 = 3x - 3$.

$4 = 2x$ and $x = 2$.

Proportions

A **proportion** is an equation that states that two ratios are equal. It is often used to solve problems involving relative sizes or rates.

Form of Proportions:

$\frac{a}{b} = \frac{c}{d}$, where $a, b, c,$ and d are numbers or expressions, and b and d are not zero.

Properties of Proportions:

1. **Cross Multiplication**: In a proportion, the product of the means equals the product of the extremes.

$$a \cdot d = b \cdot c.$$

2. **Equivalent Ratios**: Proportions can be simplified by multiplying or dividing both terms of the ratios by the same number.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{ka}{kb} = cd$ for any non-zero k.

Solving Proportions:

1. **Cross Multiplication**: Use the property of proportions to find the unknown value.

Example: Solve $\frac{2}{3} = \frac{x}{6}$.

Cross multiply: $2 \cdot 6 = 3 \cdot x$;

$12 = 3x$;

$x = 4$.

2. **Equivalent Ratios**: Simplify one ratio and compare it to the other.

Example: Solve $\frac{4}{x} = \frac{6}{9}$.

Simplify the second ratio: $\frac{4}{x} = \frac{2}{3}$;

Cross multiply: $4 \cdot 3 = 2 \cdot x$;

$12 = 2x$;

$x = 6$.

Practical Applications of Proportions:

Scaling: Use proportions to adjust the size of an object while maintaining its shape.

Example: If a map scale is $1:100$, find the actual distance for a 5 cm measurement on the map.

$\frac{1}{100} = \frac{5}{x}$;

$1 \cdot x = 100 \cdot 5$;

$x = 500$.

Rates: Use proportions to compare different rates.

Example: If a car travels 60 miles in 2 hours, how far will it travel in 5 hours?

$\frac{60}{2} = \frac{x}{5}$;

$60 \cdot 5 = 2 \cdot x$;

$x = 150$.

Vocabulary Notes

Equation - egyenlet proportion - arány mathematical statement - matematikai állítás equality - egyenlőség equals sign - egyenlőségjel solving an equation - egyenlet megoldása value - érték variable - változó true - igaz linear equations - lineáris egyenletek first degree - első fok highest power - legmagasabb hatvány general form - általános alak rational equations - racionális egyenletek ratios of polynomials - polinomok arányai radical equations – gyököt tartalmazó egyenletek roots - gyökök square roots - négyzetgyökök	cube roots - köbgyökök isolating the variable - változó izolálása cross multiplication - keresztszorzás eliminate - megszüntetni fractions - törtek proportion - arány ratios - arányok rates - arányok form - alak product of the means - belsőértékek szorzata product of the extremes -szélsőértékek szorzata equivalent ratios - egyenértékű arányok scaling - skálázás adjust the size - méret beállítása maintaining the shape - alak megőrzése map scale - rérkép méretaránya actual distance - valós távolság miles - mérföldek

Exercises

Answer the following questions on the text:

1. What is an equation in mathematical terms?
2. What does solving an equation involve?

3. What is a linear equation?
4. What is the general form of a linear equation? Give an example of a linear equation.
5. What is a quadratic equation?
6. What is the general form of a quadratic equation? Provide an example of a quadratic equation.
7. What are polynomial equations? Give an example of a polynomial equation.
8. What are rational equations? Provide an example of a rational equation.
9. What are radical equations? Provide an example of a radical equation.
10. What does isolating the variable in an equation involve?
11. Describe the process of solving the equation $5x + 3 = 23$ by isolating the variable.
12. What is factoring in the context of solving equations?
13. How can you solve the equation $x^2 - 4 = 0$ by factoring?
14. What is the quadratic formula used for, and what is its general form?
15. What is cross multiplication used for in solving equations?
16. Describe the steps to solve the equation (x+1)/(x-1)=3 using cross multiplication.
17. How do you simplify the equation x+1=3(x-1)?
18. What is a proportion in mathematical terms?
19. What is the general form of a proportion?
20. What is the property of proportions involving cross multiplication?
21. Explain the property of proportions that states the product of the means equals the product of the extremes.
22. How can you simplify proportions using equivalent ratios?
23. What is the result of multiplying or dividing both terms of a ratio by the same number?
24. How do you solve the proportion $\frac{2}{3} = \frac{x}{6}$ using cross multiplication?
25. Describe the process of simplifying the proportion $\frac{4}{x} = \frac{6}{9}$.
26. What is scaling in the context of proportions?
27. How can proportions be used to adjust the size of an object while maintaining its shape?
28. Provide an example of solving a proportion to find the actual distance on a map.
29. What is the scale factor if a map scale is $1{:}\,1000$ and a 5 cm measurement represents 3000 cm in real life?
30. How can proportions be used to compare different rates?
31. How do you calculate the distance a car travels in 5 hours if it travels 60 miles in 2 hours using proportions?

LESSON IX
RATIONAL NUMBERS

Understanding rational numbers is essential for mathematicians, as they form a fundamental part of number theory and are crucial in various mathematical operations and real-world applications.

A **rational number** is any number that can be expressed as the quotient or fraction $\frac{a}{b}$ of two integers, where a (the numerator) and b (the denominator) are integers, and $b \neq 0$. The set of rational numbers is denoted by $\mathbb{Q}$.

Properties of Rational Numbers

1. **Closure**: Rational numbers are closed under addition, subtraction, multiplication, and division (except by zero). This means that performing these operations on rational numbers will always result in another rational number.
2. **Commutativity**: Rational numbers are commutative under addition and multiplication. That is, $a + b = b + a$ and $a \times b = b \times a$ for any rational numbers a and b.
3. **Associativity**: Rational numbers are associative under addition and multiplication. That is, $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$ for any rational numbers $a, b,$ and c.
4. **Additive Identity**: The number 0 is the additive identity for rational numbers, meaning $a + 0 = a$ for any rational number a.
5. **Multiplicative Identity**: The number 1 is the multiplicative identity for rational numbers, meaning $a \times 1 = a$ for any rational number a.
6. **Additive Inverse**: Every rational number a has an additive inverse $-a$ such that $a + (-a) = 0$.
7. **Multiplicative Inverse**: Every nonzero rational number a has a multiplicative inverse $\frac{1}{a}$ such that $a \times \frac{1}{a} = 1$.

Operations with Rational Numbers

1. **Addition**: To add two rational numbers, convert them to have a common denominator and then add the numerators.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Example: $\frac{2}{3} + \frac{3}{4} = \frac{2\times4+3\times3}{3\times4} = \frac{8+9}{12} = \frac{17}{12}$.

2. **Subtraction**: To subtract two rational numbers, convert them to have a common denominator and then subtract the numerators.

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

Example: $\frac{5}{6} - \frac{1}{4} = \frac{5\times4-1\times6}{6\times4} = \frac{20-6}{24} = \frac{14}{24} = \frac{7}{12}$.

3. **Multiplication**: To multiply two rational numbers, multiply the numerators and multiply the denominators.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Example: $\frac{2}{5} \times \frac{3}{7} = \frac{2\times3}{5\times7} = \frac{6}{35}$.

4. **Division**: To divide two rational numbers, multiply the first number by the reciprocal of the second number.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Example: $\frac{3}{8} \div \frac{2}{5} = \frac{3}{8} \times \frac{5}{2} = \frac{3\times5}{8\times2} = \frac{15}{16}$.

Simplifying Rational Numbers

To simplify a rational number, divide the numerator and the denominator by their greatest common divisor (GCD). Example: Simplify $\frac{8}{12}$.

1. Find the GCD of 8 and 12, which is 4.
2. Divide both the numerator and the denominator by 4: $\frac{8\div4}{12\div4} = \frac{2}{3}$.

Converting Between Rational Numbers and Decimals

1. **From Rational Numbers to Decimals**: Divide the numerator by the denominator. Example: $\frac{7}{4} = 1.75$.
2. **From Decimals to Rational Numbers**: Write the decimal as a fraction and simplify. Example: Convert 0.75 to a rational number. $0.75 = \frac{75}{100} = \frac{3}{4}$.

Vocabulary Notes

Rational number - racionális szám real-world applications - valós alkalmazások closure - zártság additive inverse - additív inverz	multiplicative inverse - multiplikatív inverz common denominator - közös nevező greatest common divisor (GCD) - legnagyobb közös osztó (LNKO)

Exercises

I. *Answer the following questions on the text*:

1. How can a rational number be expressed?
2. What is the condition for the denominator in a rational number?
3. What does the closure property of rational numbers state?
4. Which mathematical operations are rational numbers closed under?
5. What does the commutative property of rational numbers entail?
6. What does the associative property of rational numbers state?
7. What is the additive identity for rational numbers?
8. How does the additive identity property work with rational numbers?
9. What is the multiplicative identity for rational numbers?
10. How does the multiplicative identity property work with rational numbers?
11. What is the additive inverse of a rational number a?
12. What equation shows the additive inverse property for a rational number a?
13. What is the multiplicative inverse of a nonzero rational number a?
14. What equation shows the multiplicative inverse property for a nonzero rational number a?
15. How do you add two rational numbers?
16. Solve the addition example: $2/3 + 3/4$.
17. How do you subtract two rational numbers?
18. Solve the subtraction example: $5/6 - 1/4$.
19. How do you multiply two rational numbers?
20. Solve the multiplication example: $2/5 \times 3/7$.
21. How do you divide two rational numbers?
22. Solve the division example: $3/8 \div 2/5$.
23. How do you simplify a rational number?

24. Simplify the fraction $39/26$.
25. How do you convert a rational number to a decimal?
26. Convert the fraction $9/8$ to a decimal.
27. How do you convert a decimal to a rational number?
28. Convert the decimal 0.8 to a fraction and simplify.

II. *Match each word on the left with its translation on the right.*

1. rational number	a) számláló
2. quotient	b) egész szám
3. numerator	c) reciprok
4. denominator	d) zártság
5. integer	e) összeadás
6. closure	f) nevező
7. addition	g) egyszerűsít
8. subtraction	h) asszociativitás
9. multiplication	i) szorzás
10. division	j) közös nevező
11. commutativity	k) kommutativitás
12. associativity	l) racionális szám
13. additive identity	m) legnagyobb közös osztó (LNKO)
14. multiplicative identity	n) additív egységelem
15. additive inverse	o) kivonás
16. multiplicative inverse	p) multiplikatív egységelem
17. common denominator	q) additív inverz
18. reciprocal	r) hányados
19. simplify	s) multiplikatív inverz
20. greatest common divisor (GCD)	t) osztás

LESSON X
IRRATIONAL NUMBERS

Understanding irrational numbers is vital for mathematicians, as these numbers expand the number system beyond rational numbers and play a key role in various mathematical concepts and real-world applications.

An **irrational number** is a number that cannot be expressed as a fraction $\frac{a}{b}$ of two integers, where a (the numerator) and b (the denominator) are integers and $b \neq 0$. Unlike rational numbers, irrational numbers cannot be written as a *simple fraction* and have non-repeating, non-terminating decimal expansions.

Examples of irrational numbers include π (pi), $\sqrt{2}$ (the square root of 2), and e (Euler's number).

Properties of Irrational Numbers

1. **Non-repeating, Non-terminating Decimals**: The decimal representation of an irrational number neither terminates nor repeats. For instance, the decimal expansion of π is approximately 3.14159... and continues indefinitely without repeating a pattern.
2. **Irrational Numbers and the Real Number Line**: Irrational numbers fill the gaps between rational numbers on the real number line. Together, rational and irrational numbers constitute the set of real numbers ($\mathbb{R}$).
3. **Incommensurability**: Any two irrational numbers cannot be expressed as a ratio of integers, indicating they are incommensurable. This concept dates back to the discovery of the incommensurability of the side and diagonal of a square by the ancient Greeks.

Operations with Irrational Numbers

1. **Addition and Subtraction**:

- ✓ The sum or difference of a rational number and an irrational number is irrational.
- ✓ The sum or difference of two irrational numbers can be either rational or irrational. Example: $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$ (irrational), but $\sqrt{2} - \sqrt{2} = 0$ (rational).

2. **Multiplication and Division**:

- ✓ The product or quotient of a non-zero rational number and an irrational number is irrational.
- ✓ The product or quotient of two irrational numbers can be either rational or irrational. Example: $\sqrt{2} \times \sqrt{2} = 2$ (rational), but $\pi \times 2$ (irrational).

Famous Irrational Numbers

1. **Pi ($\boldsymbol{\pi}$)**: The ratio of the circumference of a circle to its diameter. π is approximately 3.14159... and is crucial in geometry, trigonometry, and calculus.
2. **The Square Root of 2 ($\boldsymbol{\sqrt{2}}$)**: The positive solution to the equation $x^2 = 2$. It is approximately 1.41421... and was one of the first numbers proven to be irrational.
3. **Euler's Number (e)**: The base of the natural logarithm. e is approximately 2.71828... and is fundamental in calculus, particularly in the study of exponential growth and decay.

Converting Between Forms

1. **From Radical to Decimal**: Use a calculator to find a decimal approximation of a radical. Example: $\sqrt{3} \approx 1.732$.
2. **From Decimal to Radical**: Recognize familiar decimal expansions to identify possible radicals. Example: If a number is approximately 1.414, it might be $\sqrt{2}$.

Vocabulary Notes

Irrational number - irracionális szám expand - kiterjeszt beyond - túl decimal expansions - tizedes kifejtések non-repeating - nem ismétlődő non-terminating - nem végződő approximate - megközelítő indefinitely - végtelenül	real number line - valós számegyenes incommensurability - összemérhetetlenség Pi (π) - pí (π) square Root of 2 ($\sqrt{2}$) - 2 négyzetgyöke ($\sqrt{2}$) Euler's Number (e) - Euler-szám (e) radical - gyök decimal approximation - tizedes közelítés

Exercises

I. *Answer the following questions on the text*:

1. What is an irrational number?
2. How is an irrational number different from a rational number?
3. Can an irrational number be expressed as a fraction of two integers?
4. What are some examples of irrational numbers?
5. What is the decimal representation of an irrational number like?

6. What is the approximate decimal expansion of π (pi)?
7. How do irrational numbers relate to the real number line?
8. What do irrational numbers fill on the real number line?
9. What does the term "incommensurability" refer to in the context of irrational numbers?
10. Who discovered the concept of incommensurability?
11. What happens when you add a rational number and an irrational number?
12. Can the sum or difference of two irrational numbers be rational?
13. Provide an example of a rational result from the sum or difference of two irrational numbers.
14. What happens when you multiply or divide a non-zero rational number by an irrational number?
15. Can the product or quotient of two irrational numbers be rational?
16. What is the ratio of the circumference of a circle to its diameter known as?
17. What is the approximate value of the square root of 2 ($\sqrt{2}$)?
18. Why is Euler's number (e) important in calculus?
19. How can you convert a radical to a decimal approximation?

LESSON XI
DECIMAL NUMERALS

Decimal numerals, also known as base-10 numerals, form the foundation of our numerical system. They are integral to understanding mathematics, science, and everyday transactions.

Decimal numerals are based on the number ten and utilize ten unique digits: $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9. The value of each digit in a decimal numeral depends on its position, or place value, within the number. The place values increase by powers of ten from right to left.

For example, in the number $1{,}234$:

- ✓ The digit 4 is in the units place, representing 4.
- ✓ The digit 3 is in the tens place, representing 30.
- ✓ The digit 2 is in the hundreds place, representing 200.
- ✓ The digit 1 is in the thousands place, representing $1{,}000$.

To read decimal numerals, start from the leftmost digit and proceed to the right, assigning the appropriate place value to each digit. For example, $2{,}345$ is read as "two thousand three hundred forty-five."

When writing decimal numerals, align the digits according to their place values, ensuring that each digit is placed correctly to represent the intended value. Use commas to separate groups of three digits, making it easier to read large numbers.

Decimal numerals can also represent fractions using a decimal point. The decimal point separates the whole number part from the fractional part of the number. Each digit to the right of the decimal point represents a negative power of ten.

For example, in the number 12.345:

- ✓ The digit 1 is in the tens place, representing 10.
- ✓ The digit 2 is in the units place, representing 2.
- ✓ The digit 3 is in the tenths place, representing 0.3.
- ✓ The digit 4 is in the hundredths place, representing 0.04.
- ✓ The digit 5 is in the thousandths place, representing 0.005.

To convert a fraction to a decimal numeral, divide the numerator by the denominator. For example, to convert the fraction $3/4$ to a decimal numeral, divide 3 by 4 to get 0.75.

Conversely, to convert a decimal numeral to a fraction, write the decimal numeral as a fraction with a denominator that is a power of ten and simplify if possible. For example, 0.75 can be written as $75/100$, which simplifies to $3/4$.

Decimal numerals are used in various real-world applications, including:

Currency: Decimal numerals are essential for financial transactions, where amounts of money are often represented with two decimal places to indicate cents.

Measurements: Decimal numerals are used to express measurements in science and engineering, such as lengths, weights, and volumes.

Data Representation: In computing, decimal numerals are often used to represent data, such as in databases and digital systems.

Vocabulary Notes

align - igazít	negative power - negatív hatvány
applications - alkalmazások	place value - helyi érték
appropriate - megfelelő	powers of ten - tízes hatványok
assigning - hozzárendelés	proceed - folytatni
base-10 - tízesalapú	separate - elválasztani
based - alapú	unique - egyedi
commas - vesszők	various - különböző
convert - átalakít	value - érték
data - adatok	whole number part - egész szám rész

Exercises

I. *Answer the following questions on the text*:

1. On what number are decimal numerals based?
2. How many unique digits do decimal numerals utilize, and what are they?
3. What determines the value of each digit in a decimal numeral?
4. How do place values in a decimal numeral increase?
5. In the number 1,234, what is the value of the digit in the units place?
6. In the number 1,234, what is the value of the digit in the tens place?
7. In the number 1,234, what is the value of the digit in the hundreds place?
8. In the number 1,234, what is the value of the digit in the thousands place?
9. How do you read the decimal numeral 2,345?
10. When writing decimal numerals, how should digits be aligned?
11. What separates the whole number part from the fractional part in decimal numerals?
12. In the number 12.345, what does the digit in the tenths place represent?
13. How can you convert the fraction 3/4 to a decimal numeral?

II: *Say the following in English.*

1. A tizedes számok, más néven tízes alapú számok, a számrendszerünk alapját képezik.
2. A tizedes számok a tízes számrendszerre épülnek, és tíz egyedi számjegyet használnak: 0, 1, 2, 3, 4, 5, 6, 7, 8 és 9.
3. A tizedes számjegy értékét a helyi érték határozza meg, amelyben található.
4. A helyi értékek jobbról balra haladva tízes hatványokkal növekednek.
5. Például az 1234-es számnál a 4-es számjegy az egyes helyi értékben van, 4-et jelentve.

6. A 3-as számjegy a tízes helyi értékben van, 30-at jelentve.
7. A 2-es számjegy a százas helyi értékben van, 200-at jelentve.
8. Az 1-es számjegy az ezres helyi értékben van, 1000-et jelentve.
9. A tizedes számok olvasásához kezdje a legbaloldalibb számjeggyel, és haladjon jobbra, minden számjegyhez hozzárendelve a megfelelő helyi értéket.
10. Például a 2345-ös számot úgy olvassuk, hogy "kétezer-háromszáz-negyvenöt."
11. A tizedes számok írásakor a számjegyeket igazítsa a helyi értéküknek megfelelően.
12. Használjon vesszőket, hogy három számjegy csoportokat különítsen el, így könnyebben olvashatóvá téve a nagy számokat.
13. A tizedes számok tizedesvessző segítségével tört részeket is ábrázolhatnak.
14. A tizedesvessző elválasztja az egész szám részt a tört résztől.

PART II. GEOMETRY
LESSON I
GEOMETRY

Geometry is a branch of mathematics that explores the properties, measurements, and relationships of points, lines, angles, surfaces, and solids. Its applications span numerous fields, including engineering, architecture, physics, and computer graphics. Understanding geometry is fundamental to both theoretical and applied mathematics.

The origins of geometry can be traced back to ancient civilizations, including Egypt and Mesopotamia. Around 3000 BCE, the Egyptians used geometric principles to survey land, construct buildings, and develop an early form of algebra. The Mesopotamians developed techniques for measuring areas and angles, which were crucial for agriculture and astronomy.

The Greeks significantly advanced the field of geometry. Around 600 BCE, Thales of Miletus is credited with bringing Egyptian geometric knowledge to Greece. Pythagoras and his followers further developed geometric theories, including the famous Pythagorean Theorem.

Euclid, often called the "Father of Geometry," wrote The Elements around 300 BCE. This comprehensive compilation of geometric knowledge included definitions, postulates, and proofs, systematically organizing the field into a coherent whole. Euclid's work laid the foundation for classical geometry and influenced mathematical thinking for centuries.

During the Islamic Golden Age (8th to 14th centuries), scholars preserved and expanded upon Greek geometric knowledge. Mathematicians such as Al-Khwarizmi and Omar Khayyam made significant contributions to algebra and geometry. In medieval Europe, scholars like Fibonacci reintroduced and built upon this knowledge.

The Renaissance brought renewed interest in geometry, leading to developments in perspective and projective geometry in art and architecture. René Descartes and Pierre de Fermat introduced analytic geometry in the 17th century, linking algebra and geometry through the Cartesian coordinate system. In the 19th century, mathematicians such as Carl Friedrich Gauss, Nikolai Lobachevsky, and Bernhard Riemann explored non-Euclidean geometries, expanding the field's horizons.

Basic Concepts

Points

A point is an exact location in space with no dimensions—neither length, width, nor height. It is typically represented by a dot and labeled with a capital letter, for example, point A.

Lines

A line is a straight one-dimensional figure that extends infinitely in both directions. It has no thickness and is determined by two points. A line segment is part of a line that is bounded by two distinct end points. A ray is a part of a line that starts at one point and extends infinitely in one direction.

Angles

An angle is formed by two rays with a common endpoint called the vertex. Angles are measured in degrees. Types of angles include:

Acute Angle: Less than 90 degrees

Right Angle: Exactly 90 degrees

Obtuse Angle: Greater than 90 degrees but less than 180 degrees

Straight Angle: Exactly 180 degrees

Shapes and Figures

Triangles

A triangle is a polygon with three edges and three vertices. Triangles are classified based on their sides and angles:

Equilateral Triangle: All sides and angles are equal.

Isosceles Triangle: At least two sides and two angles are equal.

Scalene Triangle: All sides and angles are different.

Right Triangle: Has one right angle.

Quadrilaterals

A quadrilateral is a polygon with four edges and four vertices. Common types of quadrilaterals include:

Square: All sides are equal, and all angles are right angles.

Rectangle: Opposite sides are equal, and all angles are right angles.

Rhombus: All sides are equal, but angles are not necessarily right angles.

Parallelogram: Opposite sides are parallel and equal in length.

Trapezoid (US) / Trapezium (UK): At least one pair of opposite sides is parallel.

Circles

A circle is a set of all points in a plane that are at a given distance from a given point called the center. Important parts of a circle include:

Radius: The distance from the center to any point on the circle.

Diameter: Twice the radius, passing through the center.

Circumference: The perimeter or boundary line of a circle.

Arc: A part of the circumference.

Chord: A line segment joining two points on the circle.

Vocabulary Notes

properties - tulajdonságok points - pontok lines - egyenesek angles - szögek surfaces - felületek solids - testek applications - alkalmazások architecture - építészet computer graphics - számítógépes grafika origins - eredet ancient civilizations - ősi civilizációk Egypt - Egyiptom Mesopotamia - Mezopotámia geometric principles - geometriai elvek survey land - földmérést végez construct buildings - épületeket épít Thales of Miletus - Miletoszi Thalész Pythagoras - Pythagoras Pythagorean Theorem - Pitagorasz-tétel Euclid - Euklidesz The Elements - Elemek definitions - definíciók postulates - axiómák proofs - bizonyítások Islamic Golden Age - Az iszlám aranykor Al-Khwarizmi - Al-Khwarizmi Omar Khayyam - Omar Khayyám Medieval Europe - Középkori Európa Fibonacci - Fibonacci Renaissance - Reneszánsz	perspective - perspektíva projective geometry - projektív geometria René Descartes - Descartes Pierre de Fermat - Fermat analytic geometry - analitikus geometria coordinate system - koordináta rendszer Non-Euclidean geometries - Nem euklideszi geometriák Carl Friedrich Gauss - Gauss Nikolai Lobachevsky - Lobachevszkij triangles - háromszögek Equilateral Triangle - Egyenlőoldalú háromszög isosceles triangle - egyenlő szárú háromszög scalene triangle - Osztoptaláns háromszög (különböző oldalú) right triangle - derékszögű háromszög quadrilaterals - négyszögek square - négyzet rectangle - téglalap rhombus - rombusz parallelogram - paralelogramma trapezoid (US) / trapezium (UK) - trapéz circles - körök radius - sugár diameter - átmérő circumference - kerület src - körív chord - húr

Exercises

I. *Answer the following questions on the text*:

1. What is geometry?
2. What are some of the applications of geometry?
3. When did the study of geometry begin?
4. Which ancient civilizations made significant contributions to geometry?
5. Who is considered the 'Father of Geometry'?
6. What is The Elements by Euclid known for?
7. During which period did Islamic scholars make significant contributions to geometry?
8. What did mathematicians like Al-Khwarizmi and Omar Khayyam contribute to mathematics?
9. What was the impact of the Renaissance on geometry?

10. Who introduced analytic geometry?
11. What is the significance of the Cartesian coordinate system?
12. What are non-Euclidean geometries?
13. What is a point in geometry?
14. How is a line defined in geometry?
15. What is the difference between a line segment, a ray, and a line?
16. What is an angle, and how is it measured?
17. What are the different types of angles?
18. What is a polygon?
19. What are the different types of triangles?
20. How are quadrilaterals classified?
21. What are the key parts of a circle?

II. *Match each term on the left with its corresponding definition on the right.*

Term	Definition
1. Point	(A) A part of the circumference.
2. Line	(a) A straight one-dimensional figure that extends infinitely in both directions.
3. Line segment	
4. Ray	(B) A line segment joining two points on the circle.
5. Angle	
6. Acute angle	(b) A part of a line that starts at one point and extends infinitely in one direction.
7. Right angle	
8. Obtuse angle	(c) A part of a line that is bounded by two distinct endpoints.
9. Straight angle	(d) Formed by two rays with a common endpoint called the vertex.
10. Polygon	(e) An exact location in space with no dimensions.
11. Triangle	(f) A closed plane figure made up of straight line segments.
12. Equilateral triangle	(g) A polygon with three edges and three vertices.
13. Isosceles triangle	(h) Exactly 180 degrees.
14. Scalene triangle	(i) Greater than 90 degrees but less than 180 degrees.
15. Right triangle	(j) Exactly 90 degrees.
16. Quadrilateral	(k) Less than 90 degrees.
17. Square	(m) All sides and angles are equal.
18. Rectangle	(n) At least two sides and two angles are equal.
19. Rhombus	(o) All sides and angles are different.
20. Parallelogram	(p) Has one right angle.

21. Trapezoid 22. Circle 23. Radius 24. Diameter 25. Circumference 26. Arc 27. Chord	(q) A polygon with four edges and four vertices. (r) All sides are equal, and all angles are right angles. (s) Opposite sides are equal, and all angles are right angles. (t) All sides are equal, but angles are not necessarily right angles. (u) Opposite sides are parallel and equal in length. (v) At least one pair of opposite sides is parallel. (w) A set of all points in a plane that are at a given distance from a given point called the center. (x) The distance from the center to any point on the circle. (y) Twice the radius, passing through the center. (z) The perimeter or boundary line of a circle.

III. *Determine whether the following statements are true or false*.

1. The Egyptians used geometric principles to construct buildings around 3000 BCE.
2. Euclid is often called the "Father of Algebra."
3. A right triangle has all sides and angles equal.
4. The Cartesian coordinate system links algebra and geometry.

IV. *Draw and label the following geometric figures*:

1. An equilateral triangle
2. A right triangle
3. A square
4. A circle with its radius, diameter, and an arc labeled

V. *Look at the following descriptions and identify which type of triangle or quadrilateral is being described:*

1. A polygon with three edges where all sides are different.
2. A four-sided figure with opposite sides equal and all angles 90 degrees.
3. A triangle with one angle measuring 90 degrees.

LESSON II
BASIC FORMULAS IN PLANE GEOMETRY

In plane geometry, various formulas help calculate properties of geometric shapes and figures. These formulas are essential for solving problems related to lengths, areas, and angles. Here are some fundamental formulas:

1. **Rectangle:**

   Area: $Area = length \times width$.

   Perimeter: $Perimeter = 2 \times (length + width)$.

2. **Square:**

   Area: $Area = side^2$.

   Perimeter: $Perimeter = 4 \times side$.

3. **Triangle:**

   Area (using base and height): $Area = \frac{1}{2} \times base \times height$.

   Area (using sides): $Area = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$, where s is the semi-perimeter, $s = \frac{a+b+c}{2}$.

4. **Circle:**

   Area: $Area = \pi \times radius^2$.

   Circumference: $Circumference = 2 \times \pi \times radius$.

5. **Parallelogram:**

   Area: $Area = base \times height$.

   Perimeter: $Perimeter = 2 \times (side_1 + side_2)$.

6. **Trapezoid:**

Area: $Area = \frac{1}{2} \times (base_1 + base_2) \times height.$

7. **Regular Polygon (n-sided):**

Interior Angle: $Interior\ Angle = \frac{(n-2)\times 180°}{n}.$

Sum of Interior Angles: $Sum\ of\ Interior\ Angles = (n-2) \times 180°.$

8. **Right Triangle:**

Pythagorean Theorem: $c^2 = a^2 + b^2$, where c is the hypotenuse, and a and b are the legs.

9. **Rhombus:**

Area: $Area = \frac{1}{2} \times diagonal_1 \times diagonal_2.$

10. **Circle:**

Sector Area: $Sector\ Area = \frac{\theta}{360°} \times \pi \times radius^2$, where θ \ is the central angle of the sector.

11. **Regular Polygon (n-sided):**

Apothem: $Apothem = \frac{side\ length}{2\times tan(\frac{180°}{n})}$.

12. **Sector of a Circle:**

Arc Length: $Arc\ Length = \frac{\theta}{360°} \times 2 \times \pi \times radius.$

13. **Isosceles Trapezoid:**

Area: $Area = \frac{1}{2} \times (base_1 + base_2) \times height.$

14. **Equilateral Triangle:**

Height (altitude): $Height = \frac{\sqrt{3}}{2} \times side\ length.$

These formulas are essential for calculating areas, lengths, angles, and other properties of geometric figures, crucial for mathematical analysis and practical application in various fields of science and engineering.

Exercises

I. *Answer the following questions on the text:*

1. What branch of mathematics deals with the properties of geometric shapes and figures?
2. What kind of problems do formulas in plane geometry help solve? (e.g., lengths, areas, angles)
3. What is the formula for the area of a rectangle?
4. How do you calculate the perimeter of a rectangle?
5. What is the formula for the area of a square?
6. How do you find the perimeter of a square?
7. What formula calculates the area of a triangle using its base and height?
8. What additional information do you need to calculate the area of a triangle using its sides?
9. What is the semi-perimeter of a triangle?
10. What formula is used to find the area of a circle?
11. How do you calculate the circumference of a circle?
12. What formula calculates the area of a parallelogram?
13. How do you find the perimeter of a parallelogram?
14. What formula is used to calculate the area of a trapezoid?
15. What information do you need to know to find the interior angle of a regular polygon?
16. How do you calculate the sum of the interior angles in a regular polygon?
17. What is the Pythagorean Theorem used for in right triangles? (Identify the hypotenuse and legs)
18. What formula is used to find the area of a rhombus?
19. How can you calculate the area of a sector of a circle? (What additional information is needed?)
20. What is the formula for the apothem of a regular polygon?
21. How do you find the arc length of a sector of a circle?
22. What formula is used to calculate the area of an isosceles trapezoid?
23. How do you find the height (altitude) of an equilateral triangle?

II. *Match the following terms with their definitions*:

a) Rectangle

b) Square
c) Triangle
d) Circle
e) Parallelogram
f) Trapezoid
g) Hypotenuse
h) Leg
i) Apothem
j) Sector

Fill in the blanks with the correct word from the list above:

1) A ____ is a part of a circle enclosed by two radii and an arc.
2) The ____ are the two shorter sides of a right triangle.
3) The ____ is the perpendicular distance from the center of a regular polygon to a side.
4) The ____ is the longest side of a right triangle.
5) A ____ has one set of parallel sides.
6) A ____ has two pairs of parallel sides.
7) A ____ has four equal sides and four right angles.

III. *Problem-Solving and Formula Application*

1. A rectangular garden has a length of 15 meters and a width of 10 meters. Calculate the area and perimeter of the garden.
2. A square tile has a side length of 8 inches. Find the area and perimeter of the tile.
3. A right triangle has a base of 12 centimeters and a height of 5 centimeters. Determine the area of the triangle.
4. A circular pizza has a radius of 6 inches. Calculate the area and circumference of the pizza.
5. A parallelogram has a base of 20 centimeters and a height of 8 centimeters. What is the area of the parallelogram?
6. A trapezoid has bases of 14 meters and 18 meters, and a height of 10 meters. Calculate the area of the trapezoid.

PART III
COMPUTERS
LESSON I
THE COMPUTER HAS CHANGED OUR LIVES, EDUCATION, WORK, AND ENTERTAINMENT

Computers have revolutionized virtually every aspect of our daily lives, transforming how we live, work, and entertain ourselves. Their influence spans across various fields, impacting education, communication, healthcare, and beyond. Understanding the profound effects of computers is essential for comprehending modern society and its future directions.

Transforming Our Lives

Communication:

Email and Instant Messaging: Computers enable rapid communication through email and instant messaging services, connecting people across the globe in real time.

Social Media: Platforms like Facebook, Twitter, and Instagram allow individuals to share experiences, ideas, and updates with a vast audience instantly.

Video Conferencing: Tools like Zoom, Skype, and Microsoft Teams facilitate face-to-face communication, enhancing personal and professional interactions.

Information Access:

Internet: The internet provides an unprecedented amount of information at our fingertips, making research, learning, and staying informed easier than ever.

Online Education: E-learning platforms and online courses offer flexible and accessible education opportunities for people of all ages.

Healthcare:

Telemedicine: Computers enable remote medical consultations, improving access to healthcare for individuals in remote or underserved areas.

Medical Research: Advanced computing power allows for complex simulations and data analysis, accelerating medical research and innovation.

Revolutionizing Work

Productivity Tools:

Word Processing and Spreadsheets: Software like Microsoft Word and Excel streamline document creation, data analysis, and report generation.

Project Management: Tools like Trello, Asana, and Jira facilitate project planning, tracking, and collaboration, enhancing team productivity.

Automation:

Manufacturing: Computers control automated machinery and robotics, increasing efficiency and precision in manufacturing processes.

Administrative Tasks: Automation of repetitive tasks such as data entry and scheduling frees up human resources for more strategic activities.

Remote Work:

Flexibility: Computers enable remote work, allowing employees to work from anywhere with an internet connection, providing flexibility and work-life balance.

Collaboration: Cloud-based tools like Google Workspace and Microsoft 365 support seamless collaboration among remote teams.

Enhancing Entertainment

Gaming:

Video Games: Computers have transformed gaming into a dynamic and immersive experience with high-quality graphics, complex gameplay, and online multiplayer options.

Esports: Competitive gaming, or esports, has become a global phenomenon, with professional players, tournaments, and significant prize pools.

Streaming Services:

Movies and TV Shows: Platforms like Netflix, Amazon Prime, and Hulu offer on-demand access to a vast library of movies and TV shows, changing how we consume visual media.

Music and Podcasts: Services like Spotify, Apple Music, and various podcast platforms provide instant access to a wide range of audio content.

Creative Tools:

Digital Art and Design: Software like Adobe Photoshop and Illustrator enable artists and designers to create stunning digital artwork and graphics.

Video Editing: Tools like Adobe Premiere Pro and Final Cut Pro allow for professional-quality video production and editing.

Advancing Education

E-Learning Platforms:

Online Courses: Websites like Coursera, edX, and Khan Academy offer courses from leading universities and institutions, providing high-quality education to anyone with internet access.

Virtual Classrooms: Platforms such as Google Classroom and Blackboard facilitate virtual learning environments, allowing for real-time interaction between teachers and students.

Educational Software:

Interactive Learning: Programs like Duolingo for languages and GeoGebra for mathematics offer interactive and engaging ways to learn new subjects.

Simulations and Modeling: Software like MATLAB and Simulink allows students to perform complex simulations and modeling, enhancing their understanding of scientific and engineering concepts.

Research and Collaboration:

Digital Libraries: Access to digital libraries and academic journals through platforms like JSTOR and Google Scholar provides students and researchers with a wealth of information.

Collaboration Tools: Tools such as Mendeley and EndNote facilitate collaborative research and reference management, making group projects and academic writing more efficient.

Personalized Learning:

Adaptive Learning Technologies: Systems like Smart Sparrow and DreamBox use data and algorithms to provide personalized learning experiences tailored to individual student needs and progress.

Educational Analytics: Data analytics in education help identify learning gaps and optimize teaching strategies, improving student outcomes and engagement.

The integration of computers into every aspect of our lives, work, education, and entertainment has brought about profound changes and continues to drive innovation and progress. As computers evolve, their impact on society will only deepen, shaping the future in ways we can scarcely imagine today. Understanding these changes is crucial for anyone engaged in mathematics, technology, and related fields, as it provides insight into the tools and trends that define our modern world.

Vocabulary Notes

computer - számítógép	cloud-based tools - felhőalapú eszközök
revolutionized - forradalmasította	esports - e-sportok
communication - kommunikáció	streaming services - streaming szolgáltatások
email - e-mail	movies - filmek
instant messaging - azonnali üzenetküldés	tv shows - tévéműsorok
social media - közösségi média	music - zene
video conferencing - videokonferencia	podcasts - podcastok
information access - információhozzáférés	creative tools - kreatív eszközök
internet - internet	digital art - digitális művészet
online education - online oktatás	design - tervezés
e-learning platforms - e-learning platformok	video editing - videószerkesztés
virtual classrooms - virtuális osztálytermek	education - oktatás
healthcare - egészségügy	online courses - online kurzusok
telemedicine - távgyógyászat	interactive learning - interaktív tanulás
medical research - orvosi kutatás	simulations - szimulációk
productivity tools - termelékenységi eszközök	modeling - modellezés
word processing - szövegszerkesztés	digital libraries - digitális könyvtárak
spreadsheets - táblázatok	academic journals - tudományos folyóiratok
project management - projektmenedzsment	collaborative research - közös kutatás
automation - automatizálás	reference management - hivatkozáskezelés
manufacturing - gyártás	adaptive learning technologies - adaptív

administrative tasks - adminisztratív feladatok remote work - távmunka flexibility - rugalmasság collaboration - együttműködés	tanulási technológiák educational analytics - oktatási analitika personalized learning - személyre szabott tanulás

Exercises

I. *Answer the following questions on the text:*

1. How have computers revolutionized communication?
2. How do social media platforms like Facebook and Twitter affect our interactions?
3. What tools facilitate face-to-face communication through video conferencing?
4. How have computers improved access to healthcare in remote areas?
5. What role do computers play in medical research?
6. Which software tools are mentioned for enhancing productivity in work?
7. In what ways have computers contributed to automation in manufacturing?
8. What is the benefit of automation in administrative tasks?
9. How do computers enable remote work and provide flexibility?
10. Which cloud-based tools support collaboration among remote teams?
11. How have computers transformed the gaming experience?
12. What are some examples of online course platforms mentioned in the text?
13. How do virtual classrooms like Google Classroom and Blackboard facilitate learning?
14. What educational software is mentioned for interactive language learning?
15. How do programs like GeoGebra enhance mathematics education?
16. How do MATLAB and Simulink assist students in understanding scientific and engineering concepts?
17. What platforms provide access to digital libraries and academic journals?
18. How do tools like Mendeley and EndNote aid in collaborative research and reference management?
19. What are adaptive learning technologies, and how do they benefit students?
20. How do educational analytics help improve student outcomes?

II. *Find synonyms (words with similar meaning) from the text for the following words*:

a) Rapid (communication): ___________________
b) Unprecedented (information): _________________
c) Flexible (education): _______________________
d) Precision (manufacturing): __________________
e) Immersive (gaming): ______________________

III. *Find antonyms (words with opposite meaning) from the text for the following words*:

a) Remote (work): ___________________

b) Simple (simulations): _________________

IV. *Check (True/False) the next statement*:

a) Computers have made communication slower and more difficult. (True/False)

b) The internet provides limited access to information. (True/False)

c) Telemedicine allows doctors to treat patients remotely. (True/False)

d) Automation eliminates the need for human workers altogether. (True/False)

e) Streaming services offer a limited selection of movies and TV shows. (True/False)

V. *The text discusses various areas impacted by computers. Identify the main areas and list at least two examples of computer applications mentioned for each area*:

1. Communication:__________________, ____________________ .
2. Information Access: _____________________, __________________ .
3. Work: _____________________, __________________ .
4. Entertainment: ____________________, ___________________ .
5. Education: ________________________, _______________________.

VI. *Discuss the advantages and disadvantages of remote work*.

VII. *How have computers impacted the way you learn and access information*?

VIII. *Do you think the increasing role of computers in our lives poses any challenges? Explain your answer*.

IX. *Imagine a world without computers. Write a short story describing how daily life, communication, and entertainment would be different*.

LESSON II
COMPUTERS: THE SOFTWARE AND THE HARDWARE

Computers have become an integral part of our daily lives, revolutionizing how we work, learn, and entertain ourselves. Understanding the fundamental components of computers, both software and hardware, is essential for anyone studying or working in fields related to mathematics, science, and engineering.

Hardware

Hardware refers to the physical components of a computer that you can touch and see. These components work together to perform the basic operations required to run software and execute tasks.

Central Processing Unit (*CPU*): The CPU, often referred to as the "brain" of the computer, performs calculations and executes instructions. It consists of the arithmetic logic unit (ALU), which performs arithmetic and logical operations, and the control unit (CU), which directs the operation of the processor.

Memory (*RAM and ROM*):

Random Access Memory (*RAM*): Temporary storage that holds data and instructions that the CPU needs while performing tasks. It is volatile, meaning it loses its content when the computer is turned off.

Read-Only Memory (*ROM*): Permanent storage that contains essential instructions for booting the computer. It is non-volatile, meaning it retains its content even when the computer is off.

Storage Devices: These are used to store data permanently. Common storage devices include hard disk drives (HDDs), solid-state drives (SSDs), and optical drives (CD/DVD).

Motherboard: The main circuit board that connects all the components of the computer. It houses the CPU, memory, storage devices, and expansion slots for additional peripherals.

Input Devices: Devices that allow users to input data into the computer. Examples include keyboards, mice, scanners, and webcams.

Output Devices: Devices that allow the computer to communicate information to the user. Examples include monitors, printers, and speakers.

Graphics Processing Unit (*GPU*): A specialized processor designed to accelerate graphics rendering. GPUs are essential for gaming, video editing, and other applications requiring high-performance graphics.

Power Supply Unit (*PSU*): Converts electrical power from an outlet into a usable form for the computer's internal components.

Software

Software refers to the instructions and data that enable the hardware to perform specific tasks. There are two main categories of software: system software and application software.

System Software: Manages and controls the hardware components and provides a platform for running application software.

Operating System (*OS*): The most critical system software that manages hardware resources and provides common services for application software. Examples include Windows, macOS, Linux, and Android.

Device Drivers: Specialized software that allows the operating system to communicate with hardware devices.

Application Software: Programs designed to perform specific tasks for users.

Productivity Software: Includes word processors, spreadsheets, and presentation software (e.g., Microsoft Office, Google Workspace).

Mathematical and Scientific Software: Tools for performing mathematical calculations and scientific analysis (e.g., MATLAB, Mathematica, GeoGebra).

Graphics and Design Software: Programs for creating and editing visual content (e.g., Adobe Photoshop, AutoCAD).

Communication Software: Tools for communication and collaboration (e.g., email clients, messaging apps, video conferencing tools).

Programming Software: Tools for writing, testing, and debugging computer programs. These include text editors, compilers, and integrated development environments (IDEs) (e.g., Visual Studio, Eclipse).

The Interaction Between Hardware and Software

The interaction between hardware and software is fundamental to the operation of a computer. The hardware executes the instructions provided by the software, while the software enables users to interact with the hardware in meaningful ways.

Booting Process: When a computer is turned on, the BIOS (Basic Input/Output System) or UEFI (Unified Extensible Firmware Interface) performs initial hardware checks and loads the operating system from the storage device into RAM.

Software Execution: The CPU fetches instructions from the RAM, decodes them, and executes them. The results are then stored back in RAM or other storage devices.

Peripheral Management: Device drivers facilitate communication between the operating system and hardware peripherals, allowing for input and output operations.

Understanding the components and functions of computer hardware and software is crucial for anyone working with technology. This knowledge forms the foundation for more advanced studies in computer science, information technology, and related fields. By grasping these basic concepts, you can better appreciate the complexity and capabilities of modern computers.

Vocabulary Notes

integral part - szerves része daily lives - mindennapi élet entertain - szórakoztatni fundamental components - alapvető alkotóelemek software - szoftver hardware - hardver essential - elengedhetetlen related to - kapcsolatos physical components - fizikai alkotóelemek touch - érinteni perform - végrehajtani basic operations - alapvető műveletek required - szükséges run - futtatni execute tasks - feladatokat végrehajtani central processing unit (CPU) - központi feldolgozó egység (CPU) brain - agy calculations - számítások instructions - utasítások arithmetic logic unit (ALU) - aritmetikai logikai egység (ALU) control unit (CU) - vezérlő egység (CU) directs - irányít operation - működés processor - processzor memory (RAM and ROM) - memória (RAM és ROM) random access memory (RAM) - véletlen elérésű memória (RAM) temporary storage - ideiglenes tároló data - adatok volatile - illékony loses - elveszít	permanently - tartósan hard disk drives (HDDs) - merevlemezek (HDD-k) solid-state drives (SSDs) - szilárdtest meghajtók (SSD-k) optical drives (CD/DVD) - optikai meghajtók (CD/DVD) motherboard - alaplap main circuit board - fő áramköri lap connects - összeköt houses - befogad expansion slots - bővítőhelyek input devices - bevitel eszközök allow - lehetővé tesz input data - adatbevitel keyboards - billentyűzetek mice - egerek scanners - szkennerek webcams - webkamerák output devices - kimeneti eszközök communicate - kommunikálni monitors - monitorok printers - nyomtatók speakers - hangszórók graphics processing unit (GPU) - grafikus feldolgozó egység (GPU) accelerate - felgyorsít graphics rendering - grafikus renderelés essential for - nélkülözhetetlen gaming - játék video editing - videószerkesztés applications - alkalmazások high-performance graphics - nagy teljesítményű grafika power supply unit (PSU) - tápegység (PSU)

content - tartalom turned off - kikapcsolva read-only memory (ROM) - csak olvasható memória (ROM) permanent storage - állandó tároló booting - indítás non-volatile - nem illékony retains - megőrzi storage devices - tároló eszközök store - tárolni	electrical power - elektromos áram outlet - konnektor usable form - használható forma internal components - belső alkatrészek refers to - utal enable - lehetővé tesz perform specific tasks - konkrét feladatokat végez

Exercises

I. *Answer the following questions on the text:*

1. What is the physical part of a computer called, and why is it important?
2. What is the "brain" of the computer called, and what are its two main components?
3. What is the difference between RAM and ROM in terms of data storage?
4. List two examples of common storage devices used in computers.
5. What is the role of the motherboard in a computer system?
6. What are input devices, and give two examples?
7. How do output devices help users interact with computers? Provide two examples.
8. What is the role of a Graphics Processing Unit (GPU) in a computer?
9. What is software, and how does it differ from hardware?
10. What are the two main categories of software, and briefly explain each one?
11. What is the most critical system software, and what are some examples?
12. List two examples of productivity software commonly used in offices.
13. Give one example each of mathematical/scientific and graphics/design software.
14. Explain the booting process of a computer. What happens when you turn it on?
15. Describe the basic flow of how software interacts with hardware to execute user tasks.

II. *Match the following computer components in the left column with their descriptions in the right column.*

Components	Descriptions
1. CPU 2. RAM 3. ROM	a) Devices that allow users to provide data to the computer (keyboard, mouse). b) Devices used to permanently store data (HDD, SSD,

4. Storage Devices 5. Motherboard 6. Input Devices 7. Output Devices 8. GPU 9. Operating System 10. Application Software	CD/DVD). c) The main circuit board that connects all computer parts. d) Temporary storage that loses data when the computer is turned off. e) The "brain" of the computer that performs calculations and executes instructions. f) A specialized processor for handling graphics rendering. g) Devices that display information from the computer (monitor, printer). h) Specialized software that allows the OS to communicate with hardware. i) Programs designed to perform specific tasks for users (e.g., word processors). j) The most critical system software that manages hardware resources.

III. *Complete the following sentences with the most appropriate words from the list below.*

CPU * software * hardware * RAM * storage * monitor * output * user * instructions * data

1. The computer interprets and executes __________ provided by the software.
2. _________ refers to the physical components of a computer, such as the CPU and keyboard.
3. Information displayed on the screen is considered ___________.
4. ___________ devices allow users to interact with the computer and provide input.
5. Programs and files are stored permanently on secondary __________ devices.
6. The ___________ is the central processing unit, often referred to as the brain of the computer.
7. Computer ___________ includes both hardware and software components.
8. _____________ is temporary storage that holds data and instructions currently being used by the CPU.
9. Application software is designed to perform specific tasks for the _______________.

PART IV
OUTSTANDING MATHEMATICIANS
TEXT 1. MYKHAILO OSTROHRADSKYI

Mykhailo Ostrohradskyi (1801-1862) was a brilliant Ukrainian mathematician and physicist who made significant contributions to various fields of mathematics, including mechanics, calculus of variations, and differential geometry. His work had a profound impact on the development of these fields and continues to be studied and applied by mathematicians and scientists today.

Mykhailo Ostrohradskyi was born in the village of Pashenivka, Poltava Oblast, in present-day Ukraine, on September 24, 1801. He displayed exceptional mathematical talent from an early age, and his parents recognized his potential and enrolled him in the Poltava Gymnasium, where he excelled in his studies.

In 1820, Ostrohradskyi entered the prestigious Kharkiv University, where he further deepened his mathematical knowledge under the guidance of renowned professors. He graduated with honors in 1822, earning a master's degree in mathematics and physics.

After completing his studies, Ostrohradskyi embarked on a remarkable academic career. He taught mathematics and physics at various institutions, including Kharkiv University, the Petersburg Institute of Technology, and the Naval Academy in Saint Petersburg.

Ostrohradskyi's research spanned a wide range of mathematical topics, but he is particularly known for his contributions to mechanics and calculus of variations. He developed new methods for analyzing mechanical systems and made significant advances in the theory of optimal control problems.

Among Ostrohradskyi's most notable contributions are:

1. The Ostrohradskyi-Maxwell formula: This formula relates the moment of inertia of a rigid body to its mass distribution. It is widely used in mechanics to calculate the rotational motion of objects.
2. The Ostrohradskyi method: This method is a powerful tool for solving problems in calculus of variations. It involves introducing auxiliary variables to simplify the problem and then applying variational techniques to find the optimal solution.
3. The Ostrohradskyi-Liouville equation: This equation is a fundamental result in differential geometry that describes the curvature of surfaces. It has applications in various fields, including physics, engineering, and computer graphics.

Mykhailo Ostrohradskyi's work had a profound impact on the development of mathematics and physics. His innovative methods and groundbreaking discoveries continue to be studied and applied by researchers worldwide.

In recognition of his outstanding contributions, Ostrohradskyi received numerous honors and awards during his lifetime. He was elected a member of the Saint Petersburg Academy of Sciences in 1841 and the French Academy of Sciences in 1856.

Ostrohradskyi's legacy extends beyond his mathematical achievements. He was a passionate advocate for education and played a key role in promoting the study of mathematics and science in Ukraine. His dedication to teaching and mentorship inspired countless students and contributed to the advancement of scientific knowledge in his home country.

To honor Ostrohradskyi's memory and celebrate his remarkable contributions, several institutions and landmarks bear his name:

1. Kremenchuk Mykhailo Ostrohradskyi National University: This university in Kremenchuk, Ukraine, is one of the leading institutions of higher education in the country.
2. Ostrohradskyi Institute of Applied Mathematics and Mechanics: This research institute in Kharkiv, Ukraine, is dedicated to advancing mathematical research in areas such as mechanics, control theory, and optimization.

Mykhailo Ostrohradskyi remains an inspirational figure for mathematicians, scientists, and educators worldwide. His legacy serves as a testament to the power of human intellect and the transformative potential of mathematical exploration.

Vocabulary Notes

Brilliant - briliáns contributions - hozzájárulások mechanics - mechanika calculus of variations - variációszámítás differential geometry - differenciálgeometria profound - mélyreható development - fejlesztés studied - tanulmányozott applied - alkalmazott present-day - mai exceptional - kivételes mathematical talent - matematikai tehetség potential - tehetség enrolled - beíratott	graduated - végzett honors - kitüntetéssel master's degree - mesterfokozat embarked on - elkezdett remarkable - figyelemre méltó Naval Academy - Tengerészeti Akadémia spanned - átfogott wide range - széles skála particularly known for - különösen ismert contributions to - hozzájárulásai a/az developed - kifejlesztett new methods - új módszereket theory of optimal control problems - az optimális vezérlési problémák elméletében notable contributions - figyelemre méltó hozzájárulások

excelled - kitűnt prestigious - rangos deepened - elmélyítette mathematical knowledge - matematikai ismereteit guidance - útmutatás renowned - hírneves professors - professzorok	Ostrohradskyi-Maxwell formula - Ostrohradszkij-Maxwell-formula rigid body - merev test mass distribution - tömegeloszlás widely used - széles körben alkalmazott curvature - görbületet surfaces - felületek legacy - öröksége extends beyond - túlmutat

Exercises

I. *Answer the following questions on the text:*

1. What early sign of talent did Mykhailo Ostrohradskyi display?
2. Where did Ostrohradskyi's parents enroll him to nurture his talent?
3. What degrees did Ostrohradskyi earn at Kharkiv University?
4. Which institutions did Ostrohradskyi teach at after completing his studies?
5. What fields of mathematics is Ostrohradskyi particularly known for contributing to?
6. What honors and awards did Ostrohradskyi receive during his lifetime?
7. Which two prestigious academies elected Ostrohradskyi as a member?
8. How did Ostrohradskyi contribute to the promotion of mathematics and science in Ukraine?
9. Name two institutions or landmarks that bear Mykhailo Ostrohradskyi's name.
10. Why is Mykhailo Ostrohradskyi considered an inspirational figure for mathematicians and scientists?

II. *Find a synonym for the word "profound" in the text.*

III. *What is the antonym for the word "innovative"*?

IV. *Define the word "legacy" in your own words.*

TEXT 2. JÁNOS BOLYAI

János Bolyai, a brilliant mathematician, was born in Kolozsvár (Cluj-Napoca today) in 1802. His father, Farkas Bolyai, a mathematician himself, nurtured János's talent from a young age. Despite financial constraints, János received a strong foundation in mathematics and other sciences.

By his teens, János demonstrated exceptional abilities. He could grasp complex mathematical concepts and excelled in languages and music. However, his education was not solely focused on academics. Farkas believed in a well-rounded development, ensuring János also participated in physical activities.

János's path took an unexpected turn when his attempt to study under the renowned mathematician Carl Friedrich Gauss failed. Instead, he pursued military engineering in Vienna, graduating with honors. Though excelling in academics and sports, military life didn't suit his free spirit.

Around 1820, János embarked on a journey that would change the course of mathematics. He began questioning Euclid's parallel postulate, a fundamental axiom in geometry. Unlike his father, who initially held a different approach, János delved into non-Euclidean geometry.

By 1823, János had a breakthrough, creating a "new world" of non-Euclidean geometry. He shared his discoveries with his father, who eventually recognized their significance and encouraged him to publish. However, learning that Gauss had explored similar ideas years earlier was a major blow to János.

János's work, published as an appendix to his father's book in 1831, laid the foundation for hyperbolic geometry. Although not widely recognized during his lifetime, it is now considered a landmark achievement.

Bolyai's most famous achievement is his work on non-Euclidean geometry. Independently of the Russian mathematician Nikolai Lobachevsky, Bolyai developed a system of geometry that differed from Euclidean geometry, which had been dominant for centuries. Bolyai's work, published in 1832 as an "Appendix" to his father's work on mechanics, challenged fundamental assumptions about the nature of parallel lines and laid the groundwork for the development of hyperbolic geometry.

While Bolyai did not gain widespread recognition for his work during his lifetime, his achievements are now considered among the most important in the development of mathematics. His work on non-Euclidean geometry had a profound impact on our understanding of space, the universe, and physics.

Disillusioned and isolated, János retired from the military and lived on his family estate. He continued mathematical pursuits but remained largely unknown. He died in 1860, leaving behind a vast amount of unpublished work.

Numerous institutions and objects are named after János Bolyai, including:

- Babeş-Bolyai University in Cluj-Napoca, Romania
- János Bolyai Institute of Mathematics and Informatics in Szeged, Hungary
- The János Bolyai Prize, a Hungarian mathematics award
- Asteroid 14827 Bolyai

János Bolyai is one of the most outstanding mathematicians in history. His groundbreaking work on non-Euclidean geometry had a profound impact on our understanding of the world and laid the foundation for many modern scientific theories. His legacy continues to inspire mathematicians and scientists worldwide.

Despite the lack of immediate recognition, János Bolyai's revolutionary ideas in non-Euclidean geometry had a profound impact on mathematics and our understanding of space. His legacy continues to inspire mathematicians and scientists worldwide.

Vocabulary Notes

Grasp - Megragadni Complex - Komplex Unexpected turn - Váratlan fordulat renowned mathematician - híres matematikus embarked on a journey - útnak indult course - folyamat parallel postulate - párhuzamos axióma unlike - ellentétben delved into - elmélyedt breakthrough - áttörés "New world" - "Új világ" Non-Euclidean geometry - Nem-euklideszi geometria discovered - felfedezett shared - megosztott significance - jelentőség	encourage - bátorítani explored - kutatott similar ideas - hasonló ötletek major blow - Nagy csapás landmark achievement - megbecsült eredmény disillusioned - kiábrándult retired - nyugdíjba vonult vast amount - hatalmas mennyiség unpublished work - kiadatlan munka institutions - intézmények named after - elnevezve groundbreaking work - úttörő munka profound impact - mélyreható hatás space - tér inspire - inspirálni

Exercises

I. *Answer the following questions on the text*:

1. Who was János Bolyai's father, and what was his profession?
2. Despite financial difficulties, what kind of strong foundation did János receive in his education?
3. What did János excel at beyond academics in his teenage years?
4. Why did János Bolyai's attempt to study under Carl Friedrich Gauss fail?
5. What field of study did János pursue instead of mathematics?
6. What sparked János Bolyai's interest in non-Euclidean geometry?
7. How did János Bolyai's approach to non-Euclidean geometry differ from his father's initially?
8. What significant achievement did János Bolyai make around 1823?
9. What was a major setback for János after sharing his discoveries with his father?
10. When and how was János Bolyai's work on non-Euclidean geometry published?
11. What is the lasting impact of János Bolyai's work on non-Euclidean geometry?
12. Why did János retire from the military and live a secluded life?
13. What legacy did János Bolyai leave behind, despite not being widely recognized during his lifetime?
14. Can you name some examples of institutions or objects named after János Bolyai?

II. *Complete the following sentences using the appropriate vocabulary words*:

1. János Bolyai was a(n) _____ mathematician who made groundbreaking contributions to non-Euclidean geometry.
2. Despite _____ constraints, János received a strong education in mathematics and other sciences.
3. János demonstrated _____ abilities in mathematics and excelled in languages and music.
4. His father believed in a _____ development, ensuring János also participated in physical activities.
5. János's path took an _____ turn when he failed to study under Carl Friedrich Gauss.
6. He pursued military engineering in Vienna, graduating with _____ .
7. Around 1820, János embarked on a journey that would change the _____ of mathematics.
8. He began questioning Euclid's parallel postulate, a fundamental _____ in geometry.
9. By 1823, János had a breakthrough, creating a "new _____" of non-Euclidean geometry.
10. János's work, published as an appendix to his father's book, laid the foundation for hyperbolic geometry, a _____ achievement.

TEXT 3. JOHN VON NEUMANN

John von Neumann (born on December 28, 1903, in Budapest, Hungary, and died on February 8, 1957, in Washington, D.C., U.S.) was a Hungarian-born American mathematician. In adulthood, he added "von" to his surname, a title granted to his father in 1913. From a child prodigy, he quickly rose to be one of the leading mathematicians by his mid-twenties. His significant early work in set theory marked the beginning of a career that impacted almost every major branch of mathematics. Von Neumann's talent for applied mathematics led him to make substantial contributions to quantum theory, automata theory, economics, and defense planning. He was a pioneer in game theory and, along with Alan Turing and Claude Shannon, was one of the conceptual founders of the stored-program digital computer.

Von Neumann grew up in a wealthy, well-assimilated Jewish family. His father, Miksa Neumann (Max Neumann), was a banker, and his mother, Margit Kann (Margaret Kann), came from a successful family in the agricultural equipment business. Showing early signs of genius, von Neumann could joke in Classical Greek and memorized pages from a telephone book to recite them accurately. He learned languages and mathematics from tutors and attended the prestigious Lutheran Gymnasium in Budapest. After fleeing the short-lived communist regime of Béla Kun in 1919, his family lived in Vienna and the Adriatic resort of Abbazia (now Opatija, Croatia). Despite his father's concerns about the financial prospects in mathematics, von Neumann studied both chemistry and mathematics, earning a degree in chemical engineering from the Swiss Federal Institute in Zürich in 1925 and a doctorate in mathematics from the University of Budapest in 1926.

Von Neumann began his career during the peak influence of David Hilbert’s program of axiomatic foundations for mathematics. While still in secondary school, he wrote a paper that provided the now-standard definition of ordinal numbers, avoiding complications from Georg Cantor's transfinite numbers. His paper on the axiomatization of set theory in 1925 attracted Hilbert's attention. He worked under Hilbert at the University of Göttingen and quickly understood the implications of Kurt Gödel's incompleteness theorems, which challenged the goal of fully axiomatizing mathematics.

Von Neumann held teaching positions at the Universities of Berlin and Hamburg. His collaboration with Hilbert culminated in his book, "The Mathematical Foundations of Quantum Mechanics," which reconciled the differing quantum mechanical formulations of Erwin Schrödinger and Werner Heisenberg. His work also claimed to prove that deterministic hidden variables could not explain quantum phenomena, a conclusion that pleased Niels Bohr and Heisenberg but dismayed Albert Einstein.

By his mid-twenties, von Neumann was recognized as a wunderkind, producing pivotal papers in various fields of mathematics. In 1928, he published a key paper in game theory, focusing on the strategic element of bluffing in games like poker, distinct from the pure logic of chess or probability theory of roulette. His mini-max theorem in game theory asserted that every finite, two-person zero-sum game has a rational outcome, influencing economic and strategic decision-making.

In 1929, von Neumann was invited to lecture on quantum theory at Princeton University, leading to an appointment as a visiting professor. He was remembered as a mediocre teacher due to his fast-paced writing and erasing on the blackboard. In 1933, he became one of the first professors at the Institute for Advanced Study (IAS) in Princeton, New Jersey. He left Germany that same year as Adolf Hitler rose to power.

Von Neumann married Mariette Koevesi in 1930, and they had a daughter, Marina, who became a prominent economist. Their marriage ended in divorce in 1937, after which von Neumann married his childhood sweetheart, Klara Dan. Klara, who shared many of his interests, worked as a computer programmer.

Von Neumann introduced the theory of rings of operators, now known as von Neumann algebras, motivated by his interest in quantum phenomena. He made significant contributions to economics, earning the American Mathematical Society's Bôcher Prize in 1938.

During World War II, von Neumann worked on the Manhattan Project, applying his expertise in hydrodynamics and shock waves to the development of the atomic bomb. He contributed to the design of the Fat Man bomb dropped on Nagasaki. His collaboration with Oskar Morgenstern on "Theory of Games and Economic Behavior" in 1944 established game theory as a key tool in economics and other fields.

In the postwar years, von Neumann consulted for government and industry, contributing to the development of the ENIAC computer and establishing the architecture of the von Neumann machine. He also worked at the RAND Corporation on nuclear strategy and supported the development of the hydrogen bomb.

In his final years, von Neumann explored whether machines could reproduce themselves, anticipating later discoveries in genetics. Diagnosed with bone cancer in 1955, he continued to work until his death in 1957. Von Neumann's contributions to quantum theory, the atomic bomb, and computer science had a profound impact on the modern world, making him one of the most influential mathematicians of the 20th century.

Vocabulary Notes

applied mathematics – alkalmazott matematika atomic bomb – atombomba bluffing – blöffölés chemical engineering – vegyészmérnöki childhood – gyermekkor contributions – hozzájárulások cooperation – együttműködés decision-making – döntéshozatal development – fejlesztés doctorate – doktori fokozat early signs – korai jelek educator – oktató ENIAC computer – ENIAC számítógép expertise – szakértelem hydrodynamics – hidrodinamika Hungarian – magyar member – tag	memory – memória Nazi regime – náci rezsim nuclear strategy – nukleáris stratégia operator theory – operátorelmélet postwar years – háború utáni évek prodigy – csodagyerek quantum theory – kvantumelmélet research – kutatás secondary school – középiskola set theory – halmazelmélet significant – jelentős stored-program digital computer – tárolt programú digitális számítógép theory of rings – gyűrűelmélet tutor – oktató visiting professor – vendégprofesszor von Neumann machine – von Neumann-gép

Exercises

I. *Answer the following questions on the text*:

1. What hereditary title was added to von Neumann's surname, and when was it granted to his father?
2. At what age did von Neumann become one of the world's foremost mathematicians?
3. Which areas of mathematics did von Neumann significantly contribute to?
4. What early signs of genius did von Neumann show in his childhood?
5. Why did von Neumann's family leave Hungary in 1919, and where did they go?
6. What degrees did von Neumann earn, and from which institutions?
7. What was the significance of von Neumann's paper on transfinite ordinals?
8. Which influential mathematician did von Neumann work with at the University of Göttingen?
9. How did von Neumann contribute to the field of quantum mechanics?
10. What was the main focus of von Neumann's work in game theory?
11. Why was von Neumann invited to work on the Manhattan Project?
12. What was the design von Neumann contributed to the Fat Man atomic bomb?
13. How did von Neumann influence the development of stored-program computers?
14. In his later years, what question about machines did von Neumann explore, and what was his conclusion?

II. *Complete the following sentences using the appropriate vocabulary words from the passage*:

1. John von Neumann was a(n) _______ mathematician who made significant contributions to various fields.
2. As a child prodigy, he exhibited early signs of genius by memorizing pages from a phone book and reciting them with _______.
3. His father, concerned about the practicality of mathematics, encouraged him to pursue _______ alongside mathematics.
4. Hilbert's program focused on establishing _______ foundations for mathematics.
5. Von Neumann's work with Hilbert culminated in the book "The Mathematical Foundations of Quantum Mechanics," which reconciled the contrasting formulations of _______ and _______.
6. His paper on game theory introduced the concept of _______, a strategy involving elements of deception in games like poker.
7. After immigrating to the US, von Neumann became a professor at the prestigious _______ in New Jersey.
8. During World War II, he applied his expertise to the Manhattan Project, contributing to the design of the _______ bomb.
9. His collaboration with Oskar Morgenstern established game theory as a fundamental tool in _______ and other disciplines.
10. In the later years of his career, von Neumann explored the concept of self-replicating machines, a notion that foreshadowed discoveries in the field of _______.

III. *Match the following words with their definitions*:

1. Deterministic	(a) A person with exceptional intellectual or artistic ability at a very young age
2. Wunderkind	(b) Based on or derived from axioms
3. Pivotal	(c) To restore friendly relations between (people or groups)
4. Zero-sum	(d) To deceive an opponent into making a mistake by pretending to have something or to be able to do something that one does not have or cannot do
5. Prodigy	(e) Relating to or caused by fate or necessity; inevitable
6. Axiomatic	(f) A young person who is exceptionally talented or intelligent
7. Reconcile	(g) Of crucial importance; central
8. Bluffing	(h) Involving a situation in which one person's gain is another person's loss
9. Seminal	(i) Of moderate or average quality; not very good
10. Mediocre	(j) Influential or important in the development of something

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