ЗАКАРПАТСЬКИЙ УГОРСЬКИЙ ІНСТИТУТ ІМЕНІ ФЕРЕНЦА РАКОЦІ II
II. RÁKÓCZI FERENC KÁRPÁTALJAI MAGYAR FŐISKOLA

Кафедра математики та інформатики
Matematika és Informatika Tanszék

«Additional Topics in Contemporary Mathematics»
(Методичні вказівки для контрольних робіт)

(для студентів 2-го курсу спеціальності 014 Середня освіта (Математика))

ADDITIONAL TOPICS IN CONTEMPORARY MATHEMATICS
(Módszertani utmutató dolgozatokhoz)

Другий (магістерський) / Mesterképzés (MA)
(ступінь вищої освіти /a felsőoktatás szintje)

01 Освіта/Педагогіка */ 01 Oktatás/Pedagógia*
(галузь знань / képzési ág)

"Математика"
"Matematika"
(освітня програма / képzési program)

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Посібник з додаткових розділів сучасної математики призначений для студентів ІІ курсу (ступеня магістра) Закарпатського угорського інституту імені Ференца Ракоці спеціальності 014 Середня освіта (математика) заочної форми навчання з метою організації контрольної роботи з курсу "Додаткові розділи сучасної математики".

Матеріал призначений для використання як навчально-методичний посібник з дисципліни "Додаткові розділи сучасної математики".

Затверджено до використання у навчальному процесі
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Підготовлено до видання у електронній формі (PDF) кафедрою математики та інформатики з Видавничим відділом Закарпатського угорського інституту імені Ференца Ракоці ІІ

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За зміст посібника відповідальність несуть розробники.

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Ez a jegyzet elsősorban matematika szakos hallgatók számára készült, de hasznos lehet mindazok számára, akik bármely más szakon tanulnak matematikát.

Az oktatási folyamatban történő felhasználását jóváhagyta
a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola
Matematika és Informatika Tanszéke
(2025. június 23, 10. számú jegyzőkönyv).

Megjelentetésre javasolta a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola
Minőségbiztosítási Tanácsa
(2025. augustzus 26, 7. számú jegyzőkönyv).

Elektronikus formában (PDF fájlformátumban) történő kiadásra javasolta
a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola Tudományos Tanácsa
(2025. augustzus 28, 8. számú jegyzőkönyv).

Kiadásra előkészítette a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola
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A kiadásért felel:

DOBOS Sándor – a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola Kiadói Részlegének vezetője

A tartalomért kizárólag a jegyzet szerkesztője felel.

Kiadó: a II. Rákóczi Ferenc Kárpátaljai Magyar Főiskola (cím: 90 202, Beregszász, Kossuth tér 6. E-mail: foiskola@kmf.uz.ua)

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Variant 1

Part I. Multiple-Choice Questions

Group Theory

1. Which of the following best defines a group?
   a) A collection of numbers only
   b) A set with an operation satisfying closure, associativity, identity, and invertibility
   c) Any set with multiple operations
   d) A sequence of elements without structure

Ring Theory
2. A ring is distinguished from a group primarily by which feature?
a) The presence of two operations: addition and multiplication
b) Having only one operation
c) Requiring no identity element
d) Being finite only

Field Theory
3. What is the main property that separates a field from a ring?
a) Commutativity of addition
b) Existence of multiplicative inverses for non-zero elements
c) Closure under addition
d) Presence of a zero element

Module Theory
4. A module over a ring is best described as:
a) A ring with no addition
b) A generalization of vector spaces where scalars come from a ring
c) A set with only multiplication
d) A simple subset of a field

Galois Theory
5. Galois theory primarily studies:
a) Calculations of determinants
b) Symmetries of roots of polynomials and field extensions
c) Basic arithmetic operations
d) Matrix transformations only

Quaternion Theory
6. Quaternions extend the concept of:
a) Matrices
b) Complex numbers into four dimensions
c) Real numbers only
d) Scalars without operations

Group Ring Theory
7. A group ring combines:
a) A group and a ring structure into a single algebraic object

b) Two fields
c) Only rings with addition
d) Groups without operations

Group Representation Theory
8. The purpose of group representation is to:
a) Count elements of a group
b) Represent group elements as linear transformations or matrices
c) Compute determinants
d) Solve polynomial equations

Matrix Representations of Groups
9. Matrix representations allow:
a) Visualization of group structure through linear transformations
b) Only calculation of eigenvalues
c) Operations without linear algebra
d) Ignoring group properties

Modern Approaches in Teaching Mathematics and Informatics
10. Which of the following reflects a modern pedagogical approach?
a) Teacher-centered lectures with no interaction
b) Flipped classrooms, project-based learning, and technology integration
c) Memorization of formulas without context
d) Standardized tests only

Part II. Matching Definitions

Instructions: Match each concept (1–5) with its correct definition (A–E).

1. Problem-Based Learning (PBL)
2. Flipped Classroom
3. Computational Thinking
4. Formative Assessment
5. Interactive Learning

A. A method where students learn theoretical material outside class and work on applied tasks in class.
B. Continuous assessment aimed at monitoring learning and guiding instruction.
C. Thinking algorithmically and logically to solve problems.
D. Learning by solving real-world problems as the main instructional strategy.
E. Learning involving active student participation, collaboration, and engagement.

Part III. Fill in the Gaps (Word Bank)

Word Bank: collaboration, digital tools, critical thinking, student-centered, simulations, project-based learning, adaptive learning

1. Modern pedagogy emphasizes _______ approaches that engage students actively.
2. Virtual labs and interactive software enable exploration through _______.
3. Teaching methods aim to develop students’ _______ abilities to analyze and solve problems.

4. Incorporating computers and multimedia during lessons demonstrates effective use of _______.
5. Hands-on tasks and real-life applications illustrate _______ methods.
6. Lessons adjusting to student progress exemplify _______ strategies.
7. Group work fosters _______ among learners.

Part IV. Comparative Tasks

1. Compare **Group Theory** and **Ring Theory**: how do the structures and operations differ conceptually?
2. Compare **Field Theory** and **Module Theory**: which features are generalized in modules compared to vector spaces?
3. Compare **Galois Theory** and **Matrix Representations of Groups**: how does each theory apply to understanding symmetries and transformations?

Part V. Analytical / Reflection Questions

1. Discuss the role of **quaternions** in representing rotations in three-dimensional space and their applications in informatics or physics.
2. Analyze how **group rings** integrate the properties of groups and rings and why they are useful in algebraic structures.
3. Reflect on the significance of **group representation theory** in modern mathematics and computer science.
4. Explain how **modern pedagogical approaches** in mathematics and informatics enhance problem-solving and critical thinking skills compared to traditional methods.

Part VI. Case Study / Applied Task

1. Design a short activity applying **project-based learning** in a lecture on Ring Theory. Include objectives, student tasks, and expected outcomes.
2. Propose a scenario where **computational thinking** is integrated into a Field Theory lesson. Describe step-by-step student engagement.
3. Given a matrix representation of a group, discuss how you would explain its significance to students unfamiliar with abstract algebra.

Variant 2

Part I. Multiple-Choice Questions

Group Theory

1. Which statement best describes the concept of a subgroup?
   a) Any set of numbers
   b) A subset of a group that itself satisfies the group axioms
   c) A sequence of operations
   d) A collection of unrelated elements

Ring Theory
2. Which property distinguishes a commutative ring?
a) Multiplication is not defined
b) Multiplication is commutative for all elements
c) Addition is associative only
d) Rings must be infinite

Field Theory
3. Which feature is essential for a field but not for a general ring?
a) Existence of additive identity
b) Existence of multiplicative inverses for all nonzero elements
c) Commutativity of addition
d) Distributivity of multiplication over addition

Module Theory
4. Modules differ from vector spaces because:
a) Scalars can come from any ring, not necessarily a field
b) They have no addition operation
c) Only integers are allowed as scalars
d) They are always finite

Galois Theory
5. What is the primary goal of Galois theory?
a) To classify matrix dimensions
b) To understand the solvability of polynomials using group symmetries
c) To compute determinants
d) To simplify arithmetic operations

Quaternion Theory
6. Quaternions are primarily used to:
a) Represent four-dimensional rotations and orientations
b) Solve linear equations only
c) Replace real numbers in simple arithmetic
d) Store numerical data

Group Ring Theory
7. The main purpose of a group ring is to:
a) Merge group and ring structures for algebraic analysis
b) Define only additive operations

c) Replace matrices
d) Analyze scalar multiplication only

Group Representation Theory
8. Group representation is useful for:
a) Expressing abstract group elements as linear transformations
b) Counting polynomials
c) Evaluating derivatives
d) Studying arithmetic sequences

Matrix Representations of Groups
9. Why are matrix representations important in group theory?
a) They allow visualizing symmetries via linear transformations
b) They are only computational shortcuts
c) They replace theoretical proofs entirely
d) They simplify addition operations

Modern Approaches in Teaching Mathematics and Informatics
10. Which principle reflects a contemporary pedagogical philosophy?
a) Passive note-taking during lectures
b) Student-centered approaches with active problem-solving and collaboration
c) Memorizing definitions without application
d) Standardized testing as the sole measure of learning

Part II. Matching Concepts and Definitions

Instructions: Match each concept (1–5) with its definition (A–E).

1. Flipped Classroom
2. Formative Assessment
3. Project-Based Learning (PjBL)
4. Computational Thinking
5. Interactive Learning

A. Students solve complex tasks with real-life applications over a period of time.
B. A continuous process providing feedback and guiding students during learning.
C. Learning that combines pre-class preparation with in-class active engagement.
D. Algorithmic and analytical approach to solve problems efficiently.
E. Engaging students actively through discussions, group work, and practical exercises.

Part III. Fill in the Gaps (Word Bank)

Word Bank: abstract reasoning, engagement, technology integration, collaboration, applied learning, adaptive instruction, problem-solving

1. Modern mathematics education emphasizes _______ to connect theory with practice.
2. Students develop _______ by working on real-life tasks in lectures or labs.
3. Group activities enhance _______ among learners.
4. The use of simulations, software, and multimedia supports _______ in class.
5. Lessons designed to adjust to each student’s level exemplify _______.
6. Lectures aim to improve students’ _______ when approaching complex concepts.

7. Active participation and motivation are part of increasing student **_______**.

Part IV. Comparative / Analytical Tasks

1. Compare **Group Theory** and **Field Theory** in terms of their structural constraints and applications.
2. Compare **Ring Theory** and **Group Ring Theory**: how does the combination of structures expand algebraic possibilities?
3. Compare **Quaternion Theory** and **Matrix Representations of Groups**: how do both serve to model transformations in higher dimensions?
4. Explain how **Project-Based Learning** differs from **Flipped Classroom** approaches in fostering engagement and understanding in mathematics or informatics.

Part V. Short Essay / Reflection

1. Discuss the significance of **group representation theory** for modern computational applications.
2. Analyze the advantages and challenges of integrating **digital tools and simulations** into mathematics and informatics lessons.
3. Reflect on the importance of **critical thinking and problem-solving skills** in modern mathematical education and how different algebraic topics contribute to their development.

Part VI. Applied / Case Study

1. A teacher wants to teach **Field Theory** in a lecture with mixed-ability students. Propose a strategy using modern teaching methods, explaining steps and student tasks.
2. Design a short collaborative activity using **Group Theory** concepts, specifying objectives, roles, and outcomes.
3. Suggest how a **Matrix Representation of a Group** could be used to illustrate abstract algebra concepts to students with minimal prior exposure.

Variant 3

Part I. Multiple-Choice Questions

Group Theory

1. Which scenario illustrates the closure property in a group?
   a) Adding any two numbers sometimes yields numbers outside the set
   b) Performing the group operation on two elements always results in an element of the same set
   c) Operations are optional
   d) Elements remain unrelated under the operation

Ring Theory
2. Which statement reflects a key distinction of a non-commutative ring?
a) Addition is not defined
b) Multiplication does not necessarily commute for all elements
c) It has only one operation
d) All elements have inverses

Field Theory
3. Which combination of properties defines a field?
a) Commutative addition, associative multiplication, and no inverses
b) Associative and commutative operations, distributivity, and multiplicative inverses for nonzero elements
c) Arbitrary operations with partial closure
d) Only addition and subtraction operations

Module Theory
4. Modules extend the concept of vector spaces by:
a) Allowing scalars from rings instead of fields
b) Replacing vectors with matrices
c) Eliminating addition operations
d) Restricting scalars to integers only

Galois Theory
5. Galois theory links:
a) Graph theory and combinatorics
b) Polynomial solvability with symmetries of field extensions
c) Real number arithmetic only
d) Matrix calculations and determinants

Quaternion Theory
6. A practical application of quaternions includes:
a) Simplifying scalar addition
b) Representing 3D rotations in computer graphics and robotics
c) Evaluating polynomial roots
d) Storing only real numbers

Group Ring Theory
7. Which statement best describes a group ring?
a) It combines a group and a ring into a unified algebraic structure

b) It contains only group elements with no operations
c) It is a type of matrix
d) It generalizes fields exclusively

Group Representation Theory
8. Group representation allows:
a) Converting abstract group operations into linear transformations or matrices
b) Finding roots of polynomials
c) Simplifying arithmetic only
d) Counting elements without structure

Matrix Representations of Groups
9. How do matrix representations support understanding of group structures?
a) By providing a visual and algebraic framework for abstract symmetries
b) By replacing all theoretical work
c) By only computing scalar multiples
d) By removing the need for group axioms

Modern Approaches in Teaching Mathematics and Informatics
10. Which practice exemplifies a contemporary, student-centered approach?
a) Memorizing definitions during passive lectures
b) Integrating interactive tasks, collaborative projects, and technology
c) Strictly lecturing without questions
d) Sole reliance on standardized tests

Part II. Match the Concepts

Instructions: Connect each concept (1–5) to the correct definition (A–E).

1. Flipped Classroom
2. Problem-Based Learning (PBL)
3. Formative Assessment
4. Computational Thinking
5. Interactive Learning

A. Continuous feedback and guidance during learning processes.
B. Students actively engage in authentic problem-solving tasks.
C. Pre-class study of theory, in-class application of knowledge.
D. Logical, algorithmic approach to decompose and solve problems.
E. Learning environment where students participate in discussions and hands-on activities.

Part III. Fill in the Gaps (Word Bank)

Word Bank: collaboration, simulation, critical analysis, student-centered, adaptive strategies, applied practice, problem-solving skills

1. Modern lectures emphasize _______ approaches to encourage independent thinking.
2. Virtual environments and educational software enable learning through _______.
3. Students enhance _______ by evaluating and reasoning through complex problems.
4. Group work in lectures fosters _______ among participants.
5. Active, experiential tasks illustrate _______ in mathematics and informatics.

6. Lessons that adjust to learners' performance employ _______.
7. Real-world exercises improve _______ and understanding.

Part IV. Comparative Tasks

1. Compare **Group Theory** and **Ring Theory** in terms of operational structure and axioms.
2. Compare **Field Theory** and **Module Theory** regarding scalar sets and abstraction level.
3. Compare **Quaternion Theory** and **Matrix Representations of Groups** in their application to transformations.
4. Compare **Flipped Classroom** and **Problem-Based Learning** in fostering independent learning and engagement.

Part V. Analytical / Reflection Questions

1. Explain how **group representation theory** facilitates the connection between abstract algebra and practical computation.
2. Discuss the advantages of **integrating simulations** in teaching complex algebraic topics.
3. Reflect on the role of **critical analysis and problem-solving skills** in modern mathematics and informatics education.
4. Analyze the benefits and potential challenges of applying **Galois Theory** concepts in advanced mathematical teaching.

Part VI. Applied / Case Study

1. Design a lecture activity using **Group Ring Theory** that encourages collaborative problem-solving. Describe objectives, steps, and expected outcomes.
2. Propose an instructional scenario applying **Field Theory** concepts with adaptive strategies for mixed-ability students.
3. Describe how a **Matrix Representation of a Group** can be illustrated using computational tools for student comprehension.
4. Suggest a method for integrating **quaternions** into a practical informatics or physics lesson.

Variant 4

Part I. Multiple-Choice Questions

Group Theory

1. Which property is essential for a set and operation to form a group?
   a) Elements must be integers only
   b) Closure, associativity, identity, and inverses must hold
   c) Commutativity of multiplication is required
   d) Only finite sets are allowed

Ring Theory
2. Which of the following distinguishes a ring from a group?
a) Rings have no operations
b) Rings have two operations, typically addition and multiplication
c) Rings require only closure under multiplication
d) Rings must be finite

Field Theory
3. What characteristic is unique to fields?
a) Multiplicative inverses exist for all nonzero elements
b) Addition is associative
c) Subsets can form subfields
d) Multiplication is optional

Module Theory
4. A module generalizes vector spaces by allowing:
a) Vectors to be replaced by scalars
b) Scalars to come from a ring instead of a field
c) Only integer coefficients
d) No addition operation

Galois Theory
5. The fundamental idea of Galois theory is to:
a) Solve systems of linear equations
b) Relate polynomial solvability to group symmetries
c) Compute eigenvalues
d) Analyze matrices only

Quaternion Theory
6. Quaternions are primarily used to:
a) Extend complex numbers to represent 3D rotations
b) Simplify addition of real numbers
c) Solve linear equations only
d) Replace scalar multiplication in vector spaces

Group Ring Theory
7. A group ring is:
a) A combination of a group and a ring that forms a new algebraic structure
b) A set of matrices only

c) A ring without addition
d) A subgroup of a field

Group Representation Theory
8. Group representations allow:
a) Abstract groups to be expressed as linear transformations or matrices
b) Solving polynomials
c) Counting elements without structure
d) Simplifying arithmetic only

Matrix Representations of Groups
9. Matrix representations help in:
a) Visualizing group symmetries and transformations in linear algebraic terms
b) Replacing all theoretical reasoning
c) Performing only arithmetic calculations
d) Ignoring group axioms

Modern Approaches in Teaching Mathematics and Informatics
10. Which practice exemplifies contemporary teaching?
a) Memorization of formulas without context
b) Using student-centered, interactive, and technology-assisted learning
c) Solely teacher-centered lectures
d) Standardized testing without feedback

Part II. Matching Definitions

Instructions: Match each concept (1–5) with its correct description (A–E).

1. Flipped Classroom
2. Project-Based Learning (PjBL)
3. Formative Assessment
4. Computational Thinking
5. Interactive Learning

A. Students apply pre-learned theory in class through activities and problem-solving.
B. Students engage in real-world, complex projects as a central learning strategy.
C. Continuous evaluation to provide feedback and guide learning.
D. Structured, logical thinking to break problems into solvable steps.
E. Learning approach that involves active participation, discussion, and collaboration.

Part III. Fill in the Gaps (Word Bank)

Word Bank: collaboration, applied practice, critical thinking, adaptive learning, simulations, engagement, problem-solving

1. Modern mathematics education emphasizes _______ tasks to develop higher-order reasoning.
2. Students improve _______ by participating in interactive group exercises.
3. The use of virtual labs and modeling illustrates learning through _______.
4. Lessons that adjust to individual student progress demonstrate _______ strategies.
5. Hands-on activities in lectures foster _______ among learners.

6. Real-world exercises enhance _______ and analytical skills.
7. Active participation increases student _______.

Part IV. Comparative Tasks

1. Compare **Group Theory** and **Matrix Representations of Groups** in terms of abstraction and practical application.
2. Compare **Ring Theory** and **Group Ring Theory**: discuss how combining structures increases algebraic flexibility.
3. Compare **Field Theory** and **Module Theory**: which aspects are generalized in modules?
4. Compare **Project-Based Learning** and **Flipped Classroom**: how do they foster engagement and understanding differently?

Part V. Analytical / Reflection Questions

1. Explain the importance of **group representation theory** for connecting abstract algebra with computational applications.
2. Discuss how **quaternions** are applied in physics, computer graphics, or robotics.
3. Analyze the impact of **digital tools and simulations** on learning complex algebraic concepts.
4. Reflect on how **Galois Theory** contributes to understanding the solvability of polynomial equations.

Part VI. Applied / Case Study

1. Design an activity that integrates **Field Theory** into a mixed-ability classroom using modern teaching methods.
2. Propose a collaborative exercise for **Group Theory** to promote problem-solving and reasoning.
3. Describe how **Matrix Representations of Groups** can be visualized and taught using interactive computational tools.
4. Suggest a practical scenario where **Project-Based Learning** can be applied to teaching algebra or informatics concepts.

Variant 5

Part I. Multiple-Choice Questions

Group Theory

1. Which example best demonstrates the associative property in a group?
   a) Changing the order of operations alters results
   b) Group operations satisfy (a·b)·c = a·(b·c) for all elements
   c) Only some operations are defined
   d) Groups do not require an identity element

Ring Theory
2. Which statement describes a principal ideal domain?
a) Every ideal is generated by a single element
b) Multiplication is undefined
c) Rings cannot have additive inverses
d) Only finite sets qualify

Field Theory
3. Which property is necessary for a set to be a field?
a) Commutative and associative addition, commutative multiplication, distributive law, and multiplicative inverses for nonzero elements
b) Only associative addition
c) Multiplication may not be defined
d) Additive identity is optional

Module Theory
4. A key difference between modules and vector spaces is:
a) Modules allow scalars from arbitrary rings, not just fields
b) Modules do not include addition
c) Modules are always finite
d) Vectors must be integers

Galois Theory
5. Galois theory connects:
a) Graphs with polynomials
b) Polynomial solvability with symmetry groups of roots
c) Determinants with matrices
d) Linear equations with inequalities

Quaternion Theory
6. Quaternions extend complex numbers to:
a) Represent rotations in three and four dimensions
b) Solve linear equations only
c) Replace scalars in vector spaces exclusively
d) Simplify arithmetic operations

Group Ring Theory
7. A group ring allows:
a) Combining elements of a group with coefficients from a ring to analyze algebraic properties

b) Only scalar multiplication
c) Counting elements in a set
d) Replacing vector spaces

Group Representation Theory
8. The primary function of group representation is:
a) Expressing abstract group elements as matrices or linear transformations
b) Solving polynomial roots
c) Adding numbers only
d) Enumerating finite sets

Matrix Representations of Groups
9. Matrix representations are significant because:
a) They provide a concrete framework for analyzing group actions linearly
b) They eliminate the need for abstract algebra
c) They compute arithmetic faster
d) They replace theory entirely

Modern Approaches in Teaching Mathematics and Informatics
10. Which approach exemplifies a contemporary teaching method?
a) Passive lectures and rote memorization
b) Student-centered instruction, interactive activities, and technology integration
c) One-way teacher exposition only
d) Exclusive reliance on written exams

Part II. Match the Concepts

Instructions: Match each concept (1–5) with the correct description (A–E).

1. Flipped Classroom
2. Project-Based Learning (PBL)
3. Formative Assessment
4. Computational Thinking
5. Interactive Learning

A. Students work on structured projects to solve real-world problems.
B. Feedback is continuously used to improve learning outcomes.
C. Pre-class preparation is applied in active classroom exercises.
D. Systematic decomposition of problems into solvable steps using logic and algorithms.
E. Learning environment that emphasizes discussion, collaboration, and hands-on activities.

Part III. Fill in the Gaps (Word Bank)

Word Bank: critical thinking, collaboration, engagement, applied practice, adaptive strategies, simulations, problem-solving

1. Active tasks in lectures develop students' _______ skills.
2. Team exercises encourage _______ and communication.
3. Using educational software supports learning through _______.
4. Adjusting content and pace to student needs exemplifies _______ in teaching.
5. Real-life exercises emphasize _______ of abstract concepts.

6. Participation in interactive tasks increases student _______.
7. Lectures designed for analyzing and evaluating concepts cultivate _______.

Part IV. Comparative / Analytical Tasks

1. Compare **Group Theory** and **Ring Theory**: discuss structural complexity and practical applications.
2. Compare **Field Theory** and **Module Theory**: highlight generalizations and limitations.
3. Compare **Quaternion Theory** and **Matrix Representations of Groups** in representing transformations.
4. Compare **Project-Based Learning** and **Flipped Classroom** in promoting student autonomy and deep understanding.

Part V. Short Essays / Reflection

1. Explain how **group representation theory** bridges abstract algebra with computational applications.
2. Discuss practical uses of **quaternions** in engineering, graphics, and robotics.
3. Reflect on the advantages of **interactive and technology-assisted teaching** in modern mathematics and informatics.
4. Evaluate the relevance of **Galois Theory** for understanding the solvability of polynomials.

Part VI. Applied / Case Studies

1. Design a lecture incorporating **Field Theory** with adaptive strategies for diverse student abilities.
2. Propose a collaborative activity based on **Group Theory** concepts to enhance reasoning and problem-solving.
3. Illustrate how **Matrix Representations of Groups** can be visualized using software or interactive simulations.
4. Suggest a hands-on project applying **Modern Approaches in Teaching Mathematics and Informatics** to abstract algebra topics.

Variant 6

Part I. Multiple-Choice Questions

Group Theory

1. Which of the following illustrates the existence of an identity element in a group?
   a) An element that, when combined with any other, leaves it unchanged
   b) A group with only two elements
   c) The commutative property holds for all elements
   d) Any operation can be defined arbitrarily

Ring Theory
2. A distinguishing feature of a commutative ring is:
a) Addition is not defined
b) Multiplication is commutative for all elements
c) The ring must be finite
d) Multiplicative inverses are required for all elements

Field Theory
3. Which statement correctly describes a field?
a) Only addition and subtraction are defined
b) Both operations are commutative, associative, distributive, and nonzero elements have multiplicative inverses
c) Scalars must be integers
d) Only finite sets are allowed

Module Theory
4. How does a module differ from a vector space?
a) Scalars can be taken from a ring rather than a field
b) Modules do not allow addition of vectors
c) Modules cannot be infinite
d) Vectors are restricted to integers

Galois Theory
5. What is the central concept of Galois theory?
a) Studying the roots of polynomials through the symmetry of field extensions
b) Solving linear equations
c) Counting elements in sets
d) Evaluating determinants

Quaternion Theory
6. Quaternions are particularly useful for:
a) Representing rotations in 3D and 4D spaces
b) Simplifying addition of real numbers
c) Scalar multiplication only
d) Elementary arithmetic

Group Ring Theory
7. A group ring combines:
a) The structure of a group with coefficients from a ring to form an algebraic system

b) Only matrices
c) Counting elements
d) Scalar multiplication alone

Group Representation Theory
8. The purpose of group representation is to:
a) Express abstract group elements as linear transformations or matrices
b) Solve polynomials
c) Simplify arithmetic
d) Enumerate group elements

Matrix Representations of Groups
9. The advantage of matrix representations is:
a) They make abstract group actions tangible via linear algebra
b) They replace abstract algebra entirely
c) They compute only scalars
d) They simplify teaching trivially

Modern Approaches in Teaching Mathematics and Informatics
10. Which strategy exemplifies contemporary pedagogical methods?
a) Passive lectures with memorization
b) Student-centered, interactive, technology-assisted learning
c) Exclusive teacher-led presentations
d) Standardized testing without feedback

Part II. Match the Concepts

Instructions: Connect each concept (1–5) to its correct description (A–E).

1. Flipped Classroom
2. Project-Based Learning (PBL)
3. Formative Assessment
4. Computational Thinking
5. Interactive Learning

A. Students solve authentic problems collaboratively.
B. Pre-class study is applied in active learning sessions.
C. Provides ongoing feedback to guide student learning.
D. Systematic approach to decompose and solve problems.
E. Classroom environment emphasizes discussion, interaction, and participation.

Part III. Fill in the Gaps (Word Bank)

Word Bank: collaboration, applied practice, critical thinking, adaptive strategies, simulations, engagement, problem-solving

1. Effective lectures promote _______ among students.
2. Virtual tools and modeling foster learning through _______.
3. Assignments that adjust to student ability illustrate _______ in teaching.
4. Group exercises develop _______ and communication skills.
5. Hands-on activities reinforce _______ of abstract concepts.

6. Active participation enhances student _______.
7. Complex tasks cultivate _______ and analytical reasoning.

Part IV. Comparative / Analytical Tasks

1. Compare **Group Theory** and **Group Ring Theory**: discuss abstraction and applicability.
2. Compare **Field Theory** and **Module Theory**: highlight generalizations and practical differences.
3. Compare **Quaternion Theory** and **Matrix Representations of Groups** in representing transformations.
4. Compare **Flipped Classroom** and **Project-Based Learning** in promoting engagement, autonomy, and learning outcomes.

Part V. Short Essays / Reflection

1. Explain how **group representation theory** bridges abstract algebra and practical computation.
2. Discuss real-world applications of **quaternions** in graphics, robotics, or physics.
3. Analyze the role of **interactive and adaptive teaching methods** in mathematics and informatics.
4. Evaluate the contribution of **Galois Theory** to understanding polynomial solvability.

Part VI. Applied / Case Study

1. Design an activity integrating **Field Theory** concepts for a mixed-ability classroom using modern teaching strategies.
2. Propose a collaborative task using **Group Theory** to enhance problem-solving skills.
3. Illustrate **Matrix Representations of Groups** using interactive software or simulations.
4. Suggest a practical project applying **Modern Approaches in Teaching Mathematics and Informatics** to abstract algebra topics.

Variant 7

Part I. Multiple-Choice Questions

Group Theory

1. Which property is crucial for a set and operation to form a group?
   a) Existence of an identity element
   b) Every element must be zero
   c) Only addition is defined
   d) Associativity is optional

Ring Theory

2. In a ring, which of the following is true?
a) Multiplication is always commutative
b) Both addition and multiplication are defined and addition forms an abelian group
c) No identity is allowed
d) Rings cannot have substructures

Field Theory

3. A field differs from a ring primarily because:
a) It requires every nonzero element to have a multiplicative inverse
b) Multiplication is not defined
c) Fields must be finite sets
d) Addition is optional

Module Theory

4. Modules generalize vector spaces by allowing:
a) Scalars from arbitrary rings instead of only fields
b) Addition to be undefined
c) Vectors to be only integers
d) Multiplicative inverses for all elements

Galois Theory

5. Galois theory is primarily concerned with:
a) The connection between the structure of groups and the solvability of polynomial equations
b) Calculating determinants
c) Summing sequences
d) Solving linear systems

Quaternion Theory

6. Which statement about quaternions is correct?
a) They extend complex numbers to represent rotations in higher dimensions
b) They are strictly scalar numbers
c) They replace matrices in all contexts
d) They are one-dimensional

Group Ring Theory

7. A group ring is:
a) A combination of a ring and a group forming an algebraic structure
b) A set of only numbers

c) A tool for counting elements
d) Restricted to finite sets

Group Representation Theory
8. Group representation allows:
a) Expressing abstract group elements as matrices or linear operators
b) Replacing all algebraic operations
c) Only addition of elements
d) Eliminating group axioms

Matrix Representations of Groups
9. Why are matrix representations important in group theory?
a) They provide concrete linear forms of abstract group operations
b) They are only used for computation
c) They replace abstract algebra entirely
d) They simplify arithmetic calculations

Modern Approaches in Teaching Mathematics and Informatics
10. Which approach exemplifies modern mathematics teaching?
a) Student-centered instruction with technology and interaction
b) One-way lectures only
c) Memorization without reasoning
d) Exams as the sole evaluation method

Part II. Match the Definitions

Instructions: Match each term (1–5) to its correct description (A–E).

1. Interactive Learning
2. Formative Assessment
3. Project-Based Learning
4. Flipped Classroom
5. Computational Thinking

A. Evaluating and improving student learning continuously.
B. Classroom strategy emphasizing discussion, participation, and engagement.
C. Problem-solving using decomposition and logical steps.
D. Students work on complex projects with real-life applications.
E. Pre-class preparation applied in active sessions.

Part III. Fill in the Blanks (Word Bank)

Word Bank: critical thinking, collaboration, engagement, applied practice, adaptive methods, simulations, problem-solving

1. Real-world exercises enhance _______ skills.
2. Group tasks encourage _______ and teamwork.
3. Interactive models and software provide learning through _______.
4. Adjusting lesson plans to student needs demonstrates _______ in teaching.
5. Hands-on activities reinforce _______ of theoretical concepts.
6. Active participation increases student _______.

7. Complex exercises develop _______ abilities.

Part IV. Comparative Tasks

1. Compare **Ring Theory** and **Field Theory** in terms of structure and element properties.
2. Compare **Module Theory** and **Vector Spaces** with regard to scalar flexibility and generality.
3. Compare **Quaternion Theory** and **Matrix Representations of Groups** in handling transformations.
4. Compare **Flipped Classroom** and **Project-Based Learning** in fostering autonomy and engagement.

Part V. Short Essays / Reflection

1. Explain the role of **Group Representation Theory** in linking abstract algebra to computational applications.
2. Discuss how **Quaternions** are used in physics, engineering, and computer graphics.
3. Evaluate the impact of **interactive and technology-driven teaching methods** in mathematics education.
4. Describe the significance of **Galois Theory** in understanding polynomial equations.

Part VI. Applied / Case Study

1. Design a lecture integrating **Field Theory** concepts using adaptive teaching strategies.
2. Develop a collaborative activity based on **Group Theory** to enhance reasoning skills.
3. Illustrate **Matrix Representations of Groups** using software simulations.
4. Suggest a teaching project applying **Modern Approaches in Mathematics and Informatics** to abstract algebra topics.

Variant 8

Part I. Multiple-Choice Questions

Group Theory

1. Which of the following statements best characterizes a normal subgroup?
   a) It is invariant under conjugation by elements of the group
   b) It contains only the identity
   c) It is any subset of the group
   d) Its elements commute with every other element

Ring Theory
2. Which condition differentiates a unit in a ring?
a) It has a multiplicative inverse within the ring
b) It is zero
c) It does not interact with other elements
d) It is only additive

Field Theory
3. Which is a fundamental property of a field extension?
a) Every nonzero element of the extension has a multiplicative inverse
b) The extension is always finite
c) Addition is not required
d) Scalars are restricted to integers

Module Theory
4. A module over a ring allows:
a) Scalar multiplication using elements from a ring, not necessarily a field
b) Only integer vectors
c) Addition without scalar multiplication
d) Complete commutativity of all operations

Galois Theory
5. The Galois group of a polynomial:
a) Captures the symmetries of its roots
b) Measures the degree of a polynomial
c) Defines matrix representations
d) Counts elements in a set

Quaternion Theory
6. Quaternions differ from complex numbers because:
a) They have three distinct imaginary units
b) They are one-dimensional
c) They represent only scalars
d) They are strictly commutative

Group Ring Theory
7. A group ring is constructed by:
a) Combining a group with coefficients from a ring to form an algebraic system
b) Only enumerating group elements

c) Ignoring ring properties
d) Using matrices exclusively

Group Representation Theory
8. Which statement best describes a group representation?
a) Abstract group elements correspond to linear transformations on vector spaces
b) It solves all algebraic equations
c) It only considers scalar operations
d) It replaces groups entirely

Matrix Representations of Groups
9. Matrix representations allow:
a) Visualization of group operations through linear algebra
b) Complete elimination of abstract algebra
c) Only scalar computations
d) Ignoring group axioms

Modern Approaches in Teaching Mathematics and Informatics
10. Which practice reflects modern pedagogy in STEM education?
a) Encouraging collaborative, problem-based, and technology-assisted learning
b) One-way lectures with memorization
c) Teacher-centered instruction without feedback
d) Rigid testing as the sole assessment

Part II. Match the Definitions

Instructions: Match each concept (1–5) with its definition (A–E).

1. Adaptive Learning
2. Peer Instruction
3. Inquiry-Based Learning
4. Flipped Classroom
5. Blended Learning

A. Students learn online and in-class activities are interactive.
B. Instruction responds to individual student needs dynamically.
C. Students teach and discuss topics among peers.
D. Learning starts with questioning and problem exploration.
E. Pre-class preparation applied to in-class active work.

Part III. Fill in the Gaps (Word Bank)

Word Bank: collaboration, reasoning, engagement, simulations, problem-solving, analysis, practical application

1. Group exercises enhance _______ among students.
2. Complex problems develop analytical _______ skills.
3. Interactive models and software encourage learning through _______.
4. Applying abstract concepts to projects promotes _______.
5. Active participation fosters student _______.
6. Assignments requiring logical argumentation strengthen _______.

7. Real-world exercises improve _______ of theoretical knowledge.

Part IV. Comparative / Analytical Tasks

1. Compare **Galois Theory** and **Field Theory** in terms of abstraction and practical applications.
2. Compare **Group Representation Theory** and **Matrix Representations of Groups** in how they make abstract structures concrete.
3. Compare **Quaternion Theory** and **Matrix Representations** in modeling transformations and rotations.
4. Compare **Inquiry-Based Learning** and **Flipped Classroom** approaches in fostering independent thinking and engagement.

Part V. Short Essays / Reflection

1. Discuss how **Ring Theory** and **Group Ring Theory** complement each other in algebraic research.
2. Explain the significance of **Quaternions** in physics, engineering, or computer graphics.
3. Analyze the role of **interactive teaching and adaptive strategies** in improving student outcomes in mathematics and informatics.
4. Explain the connection between **Galois Theory** and solvability of polynomials.

Part VI. Applied / Case Study

1. Propose a classroom activity that uses **Field Theory** concepts to enhance understanding via interactive simulations.
2. Design a group exercise applying **Group Theory** to solve abstract algebra problems collaboratively.
3. Suggest a way to visualize **Matrix Representations of Groups** using software or virtual tools.
4. Create a mini-project integrating **Modern Approaches in Teaching Mathematics and Informatics** for teaching abstract algebra topics.

Variant 9

Part I. Multiple-Choice Questions

Group Theory

1. Which statement best describes a subgroup?
   a) A subset of a group closed under the group operation and inverses
   b) Any set of elements chosen randomly from the group
   c) Only the identity element
   d) A set where elements commute with all group members

Ring Theory
2. In a ring, which property is not necessarily required?
a) Commutativity of multiplication
b) Associativity of addition
c) Existence of an additive identity
d) Closure under multiplication

Field Theory
3. A finite field is distinguished from a general ring because:
a) Every nonzero element has a multiplicative inverse
b) Addition is optional
c) Multiplication is not defined
d) Elements cannot be integers

Module Theory
4. Which feature distinguishes a module from a vector space?
a) Scalars come from a ring rather than necessarily a field
b) Modules cannot have zero elements
c) Addition is undefined
d) Multiplicative inverses exist for all elements

Galois Theory
5. Galois theory connects:
a) Polynomial roots with group symmetries
b) Vector spaces with matrices
c) Ring elements with matrices
d) Linear equations only

Quaternion Theory
6. A key feature of quaternions is:
a) Non-commutativity of multiplication
b) Scalars only
c) Two imaginary units
d) One-dimensionality

Group Ring Theory
7. The combination of a group and a ring results in:
a) A group ring with additive and multiplicative structure
b) A simple matrix

c) A set of scalars only
d) A vector space

Group Representation Theory
8. Representations of groups are useful because they:
a) Translate abstract group elements into linear transformations
b) Replace rings entirely
c) Solve differential equations
d) Only apply to finite groups

Matrix Representations of Groups
9. Matrix representations help to:
a) Express abstract group operations concretely for computation
b) Avoid abstract algebra entirely
c) Represent only scalars
d) Eliminate the need for proofs

Modern Approaches in Teaching Mathematics and Informatics
10. Which strategy is considered innovative in modern pedagogy?
a) Using technology, collaboration, and student-centered approaches
b) Purely lecture-based, teacher-centered instruction
c) Memorization without reasoning
d) Only summative testing

Part II. Match the Terms

Instructions: Match the terms (1–5) with their correct definitions (A–E).

1. Problem-Based Learning
2. Adaptive Technology
3. Collaborative Learning
4. Gamification
5. Reflective Practice

A. Students engage in solving real-world, complex problems.
B. Feedback and adaptation based on student progress.
C. Students work in groups to enhance understanding.
D. Using game elements to motivate and structure learning.
E. Systematic analysis of one's own teaching and learning.

Part III. Fill in the Blanks (Word Bank)

Word Bank: analysis, synthesis, abstraction, reasoning, engagement, interactivity, modeling

1. Exercises that require drawing conclusions promote _______.
2. Applying algebraic concepts to new contexts develops _______.
3. Interactive software and simulations increase student _______.
4. Comparing different algebraic structures encourages _______ skills.
5. Understanding high-level concepts requires _______.
6. Designing new solutions from known principles enhances _______.
7. Using visual tools and simulations encourages conceptual _______.

Part IV. Comparative Tasks

1. Compare **Field Theory** and **Ring Theory** in terms of element properties and operations.
2. Compare **Group Representation Theory** and **Matrix Representations of Groups** regarding abstraction and application.
3. Compare **Quaternion Theory** and **Group Theory** in describing transformations.
4. Compare **Collaborative Learning** and **Flipped Classroom** in fostering student engagement and critical thinking.

Part V. Short Essays / Reflection

1. Explain the significance of **Galois Theory** for understanding polynomial solvability.
2. Discuss how **Quaternions** are applied in 3D rotations and computer graphics.
3. Reflect on the advantages of **interactive and technology-assisted teaching methods** in mathematics education.
4. Evaluate how **Module Theory** generalizes vector spaces and why this is important in algebra.

Part VI. Applied / Case Study

1. Propose a learning activity using **Group Theory** to encourage collaborative reasoning.
2. Develop a classroom exercise applying **Ring Theory** and **Field Theory** concepts in real-world contexts.
3. Suggest a project illustrating **Matrix Representations of Groups** using computational tools.
4. Design a lesson plan integrating **Modern Approaches in Teaching Mathematics and Informatics** to teach abstract algebra effectively.

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Variant 10

Part I. Multiple-Choice Questions

Group Theory

1. Which of the following best describes a simple group?
   a) A group with no nontrivial normal subgroups
   b) A group where all elements commute
   c) Any finite group
   d) A group consisting only of identity

Ring Theory

2. Which statement about ideals in a ring is correct?
a) An ideal absorbs multiplication by ring elements
b) Ideals must be finite sets
c) All ideals are fields
d) Ideals are subsets closed only under addition

Field Theory

3. Which condition is essential for a set with two operations to form a field?
a) Multiplicative inverses exist for all nonzero elements
b) Commutativity of addition is optional
c) Multiplication must be associative only for integers
d) Only additive inverses are required

Module Theory

4. A distinguishing feature of modules is:
a) They generalize vector spaces using a ring instead of a field
b) They require scalars from a field only
c) They cannot have substructures
d) Multiplication is always commutative

Galois Theory

5. Which is a primary application of Galois Theory?
a) Determining the solvability of polynomials by radicals
b) Representing groups as matrices
c) Defining vector spaces
d) Constructing rings from groups

Quaternion Theory

6. What is a fundamental property of quaternions?
a) Multiplication is non-commutative
b) They have only one imaginary unit
c) They are one-dimensional
d) Addition is undefined

Group Ring Theory
7. Which of the following best defines a group ring?
a) A combination of group elements and ring coefficients forming an algebraic structure
b) Only a collection of matrices
c) A field of scalars
d) A commutative vector space

Group Representation Theory
8. Why are group representations important?
a) They translate abstract group elements into linear transformations on vector spaces
b) They replace rings entirely
c) They solve algebraic equations automatically
d) They apply only to finite fields

Matrix Representations of Groups
9. Which is a main advantage of using matrix representations?
a) They allow concrete computation and visualization of group actions
b) They eliminate the need for group axioms
c) They apply only to additive operations
d) They restrict the study of algebra to finite cases

Modern Approaches in Teaching Mathematics and Informatics
10. Which approach is characteristic of modern pedagogy in mathematics?
a) Combining collaboration, inquiry, and technology-assisted learning
b) Purely lecture-based memorization
c) Teacher-centered instruction without interaction
d) Only summative assessment

Part II. Match the Definitions

Instructions: Match the terms (1–5) with their definitions (A–E).

1. Cognitive Load Theory
2. Project-Based Learning
3. Socratic Method
4. Peer Assessment
5. Blended Learning

A. Students combine online and face-to-face learning environments.
B. Evaluations are conducted by students themselves for their peers.
C. Teaching by asking guiding questions to stimulate critical thinking.
D. Designing activities where learners create tangible outputs over time.
E. Managing the mental effort required for learning complex tasks.

Part III. Fill in the Gaps (Word Bank)

Word Bank: abstraction, interaction, analysis, synthesis, reasoning, collaboration, visualization

1. Comparing different algebraic structures promotes _______.
2. Hands-on activities with software encourage student _______.

3. Combining concepts from multiple theories develops _______ skills.
4. Solving complex problems strengthens logical _______.
5. Working in groups improves _______ among learners.
6. Graphical representation of algebraic objects enhances _______.
7. Understanding high-level concepts requires _______.

Part IV. Comparative / Analytical Tasks

1. Compare **Field Theory** and **Ring Theory** in terms of element inverses and structure complexity.
2. Compare **Group Representation Theory** and **Matrix Representations of Groups** in how they enable concrete computations.
3. Compare **Quaternion Theory** and **Group Theory** for applications in rotations and transformations.
4. Compare **Project-Based Learning** and **Inquiry-Based Learning** in fostering student autonomy and problem-solving skills.

Part V. Short Essays / Reflection

1. Explain how **Galois Theory** helps understand the solvability of polynomials.
2. Discuss practical applications of **Quaternions** in engineering, physics, or computer graphics.
3. Reflect on the advantages of **collaborative and technology-assisted teaching** in mathematics and informatics.
4. Analyze how **Module Theory** generalizes vector spaces and its significance in algebraic research.

Part VI. Applied / Case Study

1. Design a classroom exercise applying **Group Theory** to collaborative problem-solving.
2. Propose a project integrating **Ring Theory** and **Field Theory** for real-world applications.
3. Suggest a way to illustrate **Matrix Representations of Groups** using computational or visualization tools.
4. Create a lesson plan employing **Modern Approaches in Teaching Mathematics and Informatics** to teach abstract algebra effectively.

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